

# Active Controller of Cantilever Beam Excited by Impact Load

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## Abstract

This work investigated the active control system of cantilever beam excited by impact external load. The control design of the system was modeled and simulated by MATLAB program for the cantilever beam using finite element technique. The controller active system used motor attached at fixed end of the beam to control the dynamic response and to give accurate results under external excitation.

The natural frequencies of the system obtained were compared with Ansys software and other research results. The comparisons shown good agreement with maximum percentage difference was (0.76)%. It was obtained from the results that the dynamic response for controlled cantilever beam exhibited better observation of the impact load, and the reduction of the overshoot was 80% approximately comparing between control and uncontrolled response for steel and aluminum beam for applied impact load. It obtained that the type of material had large effect on the vibration of the beam.

**Keywords:** Active Control Vibration, Finite Element Technique, Dynamic Excitation, Natural Frequencies, Tip External Load.

## الخلاصة

البحث الحالي تناول نظام السيطرة الفعالة للعتبة المسندة من طرف واحد بتأثير صدمة ناتجة من حمل خارجي. كانت نمذجة ومحاكاة تصميم السيطرة على النظام بواسطة برنامج MATLAB وذلك باستخدام تقنية العناصر المحددة. النظام الفعال المسيطر استخدم ماطور مربوط على النهاية المسندة للعتبة للسيطرة على الاستجابة الديناميكية ولإعطاء نتائج دقيقة بتأثير الأحمال الخارجية. الترددات الطبيعية للنظام و المستحصلة من التحليل قورنت مع النتائج المستحصلة من برنامج Ansys وكذلك مع النتائج المستحصلة بواسطة بحث آخر. بينت المقارنات توافق جيد مع نسبة خطأ لا تتجاوز 0.76 بالمائة. تم الحصول من نتائج هذه الدراسة على انه الاستجابة الديناميكية للعتبة المسندة من طرف واحد و المسيطر أظهرت تخميد أفضل للصدمة الناتجة من الحمل الخارجي، وكان النقصان لأعلى قيمه في هذه الاستجابة بما يقارب 80% مقارنة ما بين بوجود وعدم وجود السيطرة للاستجابة في حالة أعتبه المصنوعة من الحديد و المصنوعة من الألمنيوم للأحمال المسلطة الخارجية. وجد لنوع المعدن تأثير كبير على اهتزاز أعتبه. **الكلمات المفتاحية :** فعالية سيطرة الاهتزاز، تقنية العناصر المحددة، الإثارة الديناميكية، الترددات الطبيعية، الحمل الخارجي للنهاية الحرة.

## Introduction

Beams represent the fundamental models for the structural elements of many engineering applications and they had been studied extensively. In engineering there are many examples of the structures that can be modeled with beam such as tall buildings, long span bridges, and arms of robot (Gawali and Sanjay, 2011).

The body vibrated under influence of external load can be considered under forced vibration, for example: vibration in machines. The vibration can be undesirable for structures and machines because it may be increased the produced stresses, bearing loads, energy losses, induce fatigue, cause added wear and absorb energy from the system. When the natural frequency is equaled to external exciting frequency; the vibration produced has same frequency for the applied force; then the resonance will occur in the system (Kamble *et.al.*, 2016).

The properties of the vibration for the engineering devices are limiting factors of their efficiency, and the vibration may be harmful and must be avoided. The control vibration can used for many applications such as aircraft structures, space structures, helicopters, civil engineering structures, wind tunnel stings and machine tools. In all these situations can be isolated from the source of the vibrations. Automobiles has

absorber shock to isolate the body and passengers from road roughness, and the energy are dissipated by the damping (Prashant and Gangadharan, 2012).

The finite element technique proved to be a powerful method of complicated structures analysis. The influence of active controller depends on the control strategy and the mathematical model of system or structure (Najeeb and Naushad, 2012).

(Alexandre *et.al.*, 2010) presented the optimal design of structural for cantilever beam through a method of the homogenization design. The force was determined by the optimization of the control design for dynamic response and implement by state space modal. In this case, the locations of the actuators were chosen arbitrarily prior to the structural design. The authors obtained that the best location of the actuator for cantilever beam near to the fixed end.

(Ronaldo *et.al.*, 2012) studied three techniques of the control: the active, the passive and the semi-active control. Proposed an Euler Bernoulli beam model and incorporated the piezoelectric sensor and actuator dynamic using the finite element method. There were multi factors took into consideration, among them the type of sensor and actuator being used. This study used a material with a piezoelectric properties which acted both as sensor and actuator.

(Najeeb and Naushad, 2012) investigated the vibration suppression of smart beams. Experimental analysis to determine the controlling of various beams of different dimensions has been used to show the effective. The sensor and actuator calculations for various cantilever beams may be used like a benchmark for analytical study. The experimental results shown that the better compatibility for the calculating results by finite element Abaqus method based on the theory of the zigzag.

In (Saurabh *et.al.*, 2014) the modeling and design of the beam based on two piezoelectric ceramic lead zirconate titanate (PZT) patches has been attached for the upper and lower surface as sensor-actuator pair. The cantilever Euler Bernoulli beam has been considered. It shown that the response without control was essential, and when applying control effects a suitable vibration damping had been done. It obtained that the accurate result acquired when the Piezoelectric patches attached nearby the fixed end.

(Kamble *et.al.*, 2016) dealt with the vibration of the cantilever beam occur due to external Force. The analytical and experimental methods to finding the natural frequency of different modes for two materials (Aluminum and Mild Steel) have been used. The researchers concluded that the frequencies Aluminum were greater than Mild Steel for same Mode Shapes. Also when the number of modes increase, the difference between the analytical and experimental frequencies reduce and close up to same value. Also, If the number of modes increased, then the acceleration of the beam will also increase.

The object of this paper is presented to estimate an active and passive effect on the cantilever beam vibration due to external impact load subjected on the free end. The dynamic model of the beam is derived and then solved the problem using finite element method by Matlab program. Different types of materials are used to determine the difference in the beam vibration for each type of materials.

## **Mathematical Model and Finite Element Formulation**

The beam is considered as Euler Bernoulli beam elements. i.e. transverse shear forces effect is neglected, and cross section of the beam remain plane and perpendicular to the deformed longitudinal axis before and after bending. The node undergoes both translational ( $u_1, u_2$ ) and rotational displacements ( $\theta_1, \theta_2$ ). The linear forces are  $f_1$  and  $f_2$  corresponding to linear displacements ( $u_1, u_2$ ) and rotational joint

loads; i.e. bending moments;  $M_1$  and  $M_2$  are corresponding to the rotational displacements ( $\theta_1, \theta_2$ ). The two nodes finite element of a regular element for the beam is described in figure (1).

The beam element has the shape function can be obtained as (Saurabh et.al., 2014).

$$N(x) = \left[ 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad x - \frac{2x^2}{l} + \frac{x^3}{l^2} \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \frac{-x^2}{l} + \frac{x^3}{l^2} \right] \quad (1)$$

where  $l$  is the elemental length. The nodal displacement can be written as.

$$\{q\}^T = [u_1 \theta_1 u_2 \theta_2] \quad (2)$$

The Lagrange's equations gives the Kinetic energy and Potential energy respectively of the structure as.

$$T = \frac{1}{2} \{\dot{q}\}^T [m] \{\dot{q}\} \quad (3)$$

$$U = \frac{1}{2} \{q\}^T [k] \{q\} \quad (4)$$

Where:

$T$  is the kinetic energy.

$U$  is the potential energy.

$[m]$  mass matrix.

$[k]$  stiffness matrix.

And  $\{q\}$  displacement vector (generalized coordinates).

The kinetic energy and potential energy can be produced the mass matrix and element stiffness matrix of the beam as (Saurabh et.al., 2014).

$$[m] = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & -54 & -13l \\ 22l & 4l^2 & -13l & -3l^2 \\ 54 & 13l & -156 & -22l \\ -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \quad (5)$$

$$[k] = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (6)$$

Where:

$\rho$  is material density,  $A$  is beam cross section area and  $l$  is the length of the element.

Therefore the elemental mass and stiffness matrices can be calculated of the ( $n$ th) element as (Tokhil et.al., 1999).

$$[m] = \frac{\rho A l}{420} \begin{bmatrix} 140l(3n^2 - 3n + 1) & 21(10n - 7) & 7l(5n - 3) & 21(10n - 3) & -7l(5n - 2) \\ 21(10n - 7) & 156 & 22l & -54 & -13l \\ 7l(5n - 3) & 22l & 4l^2 & -13l & -3l^2 \\ 21(10n - 3) & 54 & 13l & -156 & -22l \\ -7l(5n - 2) & -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \quad (7)$$

$$[k] = \frac{EI}{l^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6l & -12 & 6l \\ 0 & 6l & 4l^2 & -6l & 2l^2 \\ 0 & -12 & -6l & 12 & -6l \\ 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (8)$$

The damping matrix  $[D]$  is presented as follows [Ronaldo et. al., 2012].

$$[D] = \alpha [m] + \beta [k] \quad (9)$$

Where:  $\alpha=0.001$  and  $\beta=0.0001$  are the frictional damping constant. Note that elemental mass matrix is dependent on elements number, and the elemental stiffness matrix gives the same value of  $n$ th elements. These matrices have been

calculated and used to obtain the dynamic response of the beam as ( Tokhil *et.al.*, 1999 ).

$$[m]\{\ddot{q}\} + [k]\{q\} = \{F(t)\} \quad (10)$$

Where  $\{\ddot{q}\}$  is the acceleration vector and  $\{F(t)\}$  is external forces vector.

### Dynamic Equation of the Structure

To including the controller effect in equation (10), it may be written as [Umapathy and Bandyopadhyay, 2000].

$$[m]\{\ddot{q}\} + [k]\{q\} = \{F(t)\} + \{F_{Actu}\} \quad (11)$$

Where  $\{F_{Actu}\}$  is the actuator forces vector. To solve and formulate the equation (11) for structure by using the transformation  $\{q\} = [\psi]\{x\}$  for minimizing the equation of motion (11), in order to represents the dynamics of first two dominant vibratory modes of the system. Where  $[\psi]$  is the modal matrix containing the eigenvectors and  $\{x\}$  is the principal coordinates. Then equation (11) becomes.

$$[m^r]\{\ddot{x}\} + [k^r]\{x\} = \{F^r(t)\} + \{F^r_{Actu}\} \quad (12)$$

If the damping effect is introduce by.

$$[D^r] = \alpha[m^r] + \beta[k^r] \quad (13)$$

The dynamic equation of the cantilever beam developed is obtained as.

$$[m^r]\{\ddot{x}\} + [D^r]\{\dot{x}\} + [k^r]\{x\} = \{F^r(t)\} + \{F^r_{Actu}\} \quad (14)$$

### State Space Modal of the Structure

The state space model of the cantilever beam can obtained by assuming  $\{x\} = \{y\}$  relation as (Manjunath and Bandyopadhyay, 2006).

$$\{x\} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \{y\} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (15)$$

And assume other assumption as.

$$\{\dot{x}\} = \{\dot{y}\} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} y_3 \\ y_4 \end{bmatrix} \text{ and } \{\ddot{x}\} = \{\ddot{y}\} = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} \quad (16)$$

Equation (14) may resulted as.

$$[m^r] \begin{bmatrix} \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} + [D^r] \begin{bmatrix} y_3 \\ y_4 \end{bmatrix} + [k^r] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \{F^r(t)\} + \{F^r_{Actu}\} \quad (17)$$

This Equation can be simplified as.

$$\begin{bmatrix} \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = -[m^r]^{-1}[k^r] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - [m^r]^{-1}[D^r] \begin{bmatrix} y_3 \\ y_4 \end{bmatrix} + [m^r]^{-1}\{F^r(t)\} + [m^r]^{-1}\{F^r_{Actu}\} \quad (18)$$

The state space equation form of the beam can be written from equation (18) as.

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0 & [I] \\ -[m^r]^{-1}[k^r] & -[m^r]^{-1}[D^r] \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} \{0\} \\ [m^r]^{-1}[\psi]^T\{f\} \end{bmatrix} u(t) \quad (19)$$

Where  $[I]$  is the unit matrix,  $u(t)$  is magnitude of external load and  $\{f\}$  is unit force vector. The state space model of a beam for the first two dominant vibratory modes is written as.

$$\{\dot{y}\} = [A]\{y(t)\} + [B]u(t) \quad (20)$$

Where

$$[A] = \begin{bmatrix} 0 & [I] \\ -[m^r]^{-1}[k^r] & -[m^r]^{-1}[D^r] \end{bmatrix} \text{ and } [B] = \begin{bmatrix} \{0\} \\ [m^r]^{-1}[\psi]^T\{f\} \end{bmatrix}$$

### Controller System Description

The block diagram of the vibration beam control with feedback used in this study can be shown in figure (2). In this work, a motor fixed near to the supported end of the beam is used to generate the required torque for controlling the dynamic

response of the beam. This because of the recent studies showed that the best result of control vibration for the structure due to the controller location near the fixed end (Alexandre *et.al.*, 2010 ; Saurabh *et.al.*,2014).

The modeling and controlling techniques for the structures in earlier time depended on the modern control theory such as state and output feedback to design the controller system, where the state feedback controller with these control techniques needs an estimator or the availability of the entire state vector ( Manjunath and Bandyopadhyay, 2006).

The basic control methods; for example root locus; depended on a simple input output description of the plant, commonly called the transfer function of the plant. The modern control theories solve a number of restrictions by using a much "richer" description of the dynamic plant. The state space description gives the dynamic model as a set of first order differential equations with respect to internal variables "known as state variables" together with a set of algebraic equation that combined the state variables into physically output variables. The aim is to gets feedback with high gain at the natural frequency. The controller can be down the resonant peaks of the vibrating structure while having only restricted effect at other frequencies (Hendra *et. al.*, 2004).

### Modal Analysis of the Structure

The finite element methods are the numerical procedures for solving physical problems administered by differential equations. They have two characteristics that recognize it from the numerical procedures[Ronaldo *et. al.*, 2012]:

- 1) used an integral formulation to create a system with algebraic equations.
- 2) utilize a continuous piecewise with smooth functions for approximating the undetermined quantities.

To estimate eigenvalue of the system, the external load in equation (10) will be equal to zero such as.

$$[m]\{\ddot{q}\} + [k]\{q\} = 0 \quad (21)$$

While the movement of the free vibration is a simple harmonic equation, the solution will be [Ronaldo *et. al.*, 2012].

$$q(t) = A \sin(\omega t) \quad (22)$$

Where A is the amplitude,  $\omega$  is the natural frequency and t is the time. Substituting (22) in (21), it is possible to give as.

$$[[k] - \omega^2[m]]\{q\} = 0 \quad (23)$$

Equation (23) represents a problem of characteristic value or eigenvalue. The value of  $\omega^2$  are eigenvalues refer to the square root of the natural frequencies and the corresponding values of vector  $\{q\}$  refers to the eigenvectors or the mode shapes of the system.

### Comparative Study

In this subsection, a comparative study has performed to validate the present results. The FEM results were compared with the results calculated using a suitable software such as Ansys (version 14).Also, the present results were compared with the results obtained by (Mehmet Avcar, 2014). These compressions shown in table (1).

**Table (1) Comparing present work results with others for cantilever properties of (E=70 GPa,  $\rho=2700\text{kg/m}^3$ , L=3m, A=0.04m<sup>2</sup>).**

mode	FEM result	Ansys	Error (%)	( Mehmet Avcar, 2014)	Error (%)
1	18.4	18.26	0.7608	18.5	0.5405
2	114.6	113.87	0.6370	115.3	0.6071
3	321.89	320.4	0.4628	322.5	0.1891

As shown from the table (1), the structure frequencies and percentage error. The present results of this study shown good comparisons and may be progress the analysis.

## Results and Discussions

Considering a cantilever beam bonded with actuator motor. It can be used two cases for the beam material to study the controller effect of vibration, there are steel and aluminum beam. And the beam loaded with external disturbing impact force (**P**) is acting at the free end. Table (2) shows the material properties and the applied load value for each type of materials used in this analysis. Figure (3) shows the chosen beam to simulate the object of this study and position of the actuator motor.

The beam is divided into 5 finite elements, where the more elements number don't have large effect on accuracy of the results as shown in table (3), where increasing the number of elements gave the percentage error was less than (0.4 %). And a higher number of elements can be represent a time consuming simulation problem.

**Table (2) Material properties and applied load.**

material	Modulus of elasticity (GPa)	Density (kg/m <sup>3</sup> )	Impact load (N) applied at cantilever tip for (0.01 sec)
Steel	210	7850	10
Aluminum	70	2700	1

**Table (3) The natural frequencies using different number of elements**

Mode	Present work with 5 elements	Ansysis with 10 elements	Error%	Ansysis with 15 elements	Error%
1	18.4	18.34	0.33	18.38	0.11
2	114.6	114.43	0.15	114.23	0.32
3	321.89	320.65	0.38	320.89	0.31

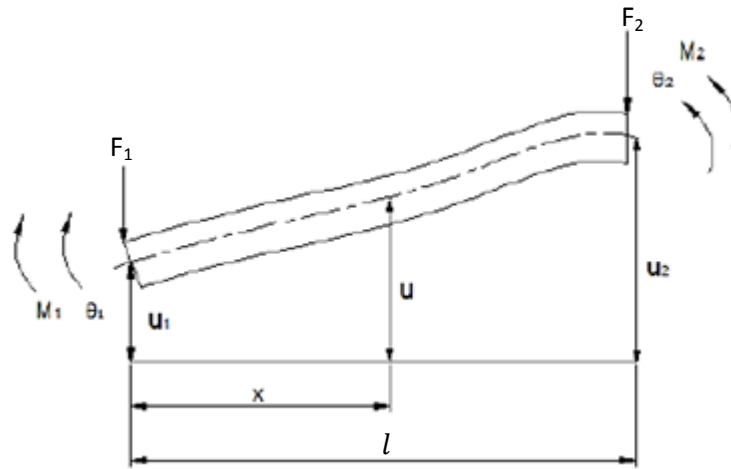
From the recent studies, it is a known that a better location for an actuator along the cantilever beam is near the fixed end of the beam, since it effect upon the first and most important mode. The design of the current study via open and closed loops for passive and active controller vibration using MATLAB program has been evaluated.

The torque of the motors shown in figures (4) and (5) for steel and aluminum beam respectively. And the system dynamic response with passive and active control action for the impulse input with time domain may be described in the figures (6) and (7) for steel beam and aluminum beam respectively.

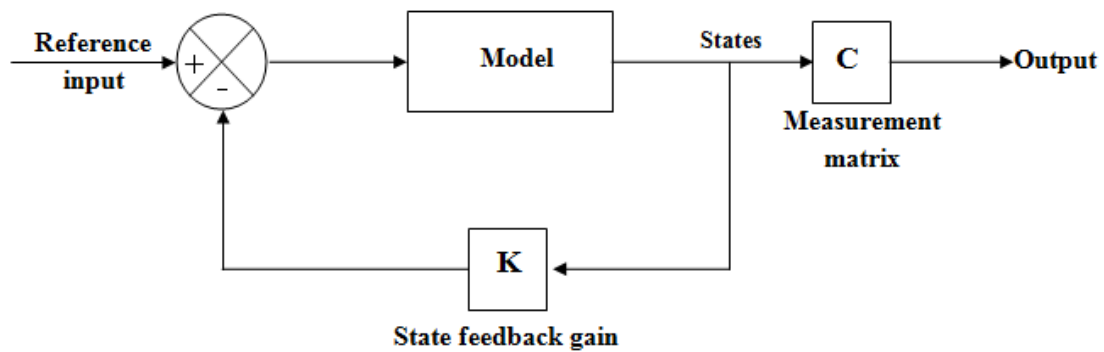
The system without controller has large response for excitation by impulse load and this response tend to reduce when applied the controller system. This means that without controller the excitation taken large effect on the beam. While the obtained results shown that the cyclic addressing of excitation taken lowest effect on the beam with controller.

From figures (6) and (7) can seen that the type of material had large effect on the vibration of the beam, and this idea required different torque value of the motor for the applied load to control on the beam vibration in each type of material as shown in previous figures. It showed that the reduction of the beam displacement for active controller is very clear. Where the figures shown the overshoot and settling time tended to minimum value after applied the torque of motor. The overshoot reduced by 80% approximately for active case with respect of passive case. This mean that the motor torque control by the damping ratio of whole system when dragging the load

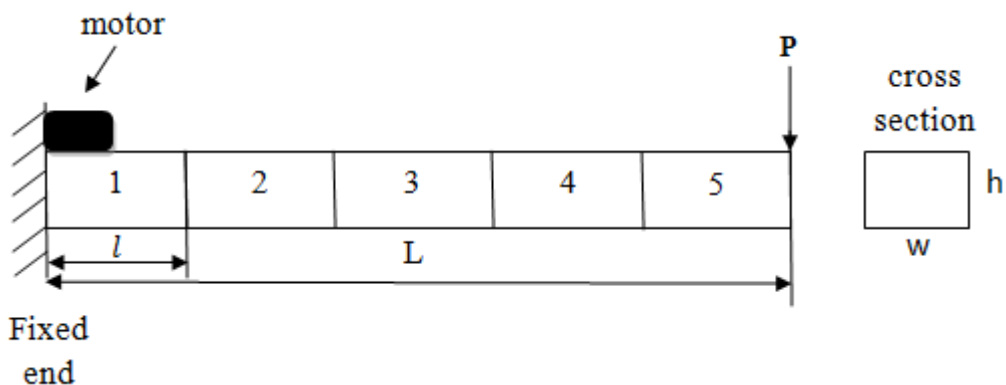
excitation was exist. The settling time of the response don't take more than (4 sec) period for steel and (2 sec) period for aluminum to stabilize, after this period the controller effect became approximately negligible.



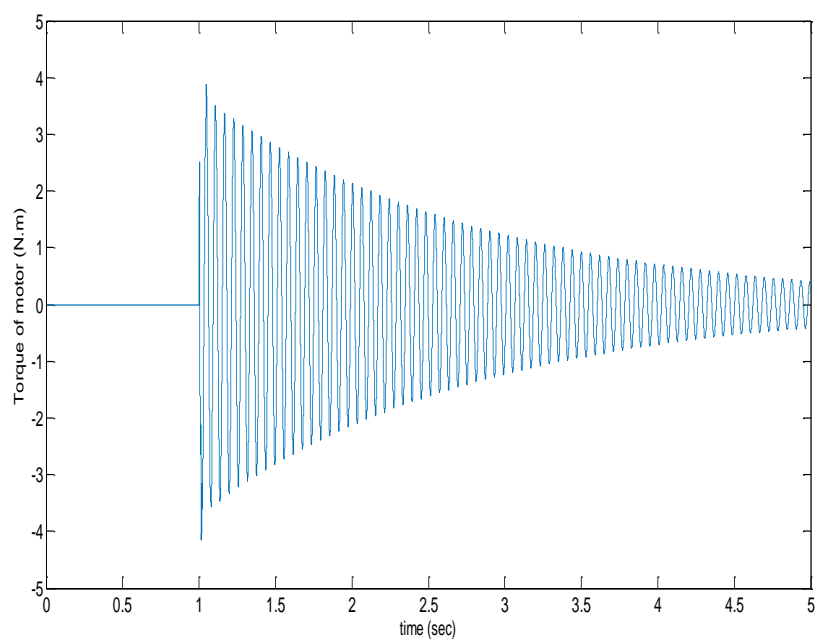
**Fig.1 : Two nodes element of a regular element for the beam (Saurabh *et.al.*, 2014).**



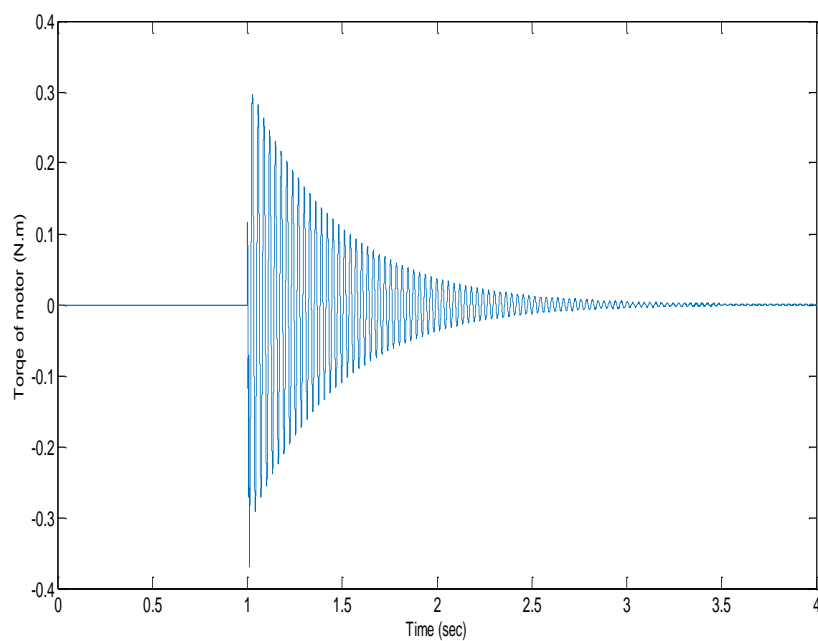
**Fig. ( 2 ) Block diagram for control of the beam.**



**Fig. ( 3 ) Cantilever beam model used in the simulation ( $w=0.06$  m,  $h=0.01$  m and  $L=0.5$ m).**

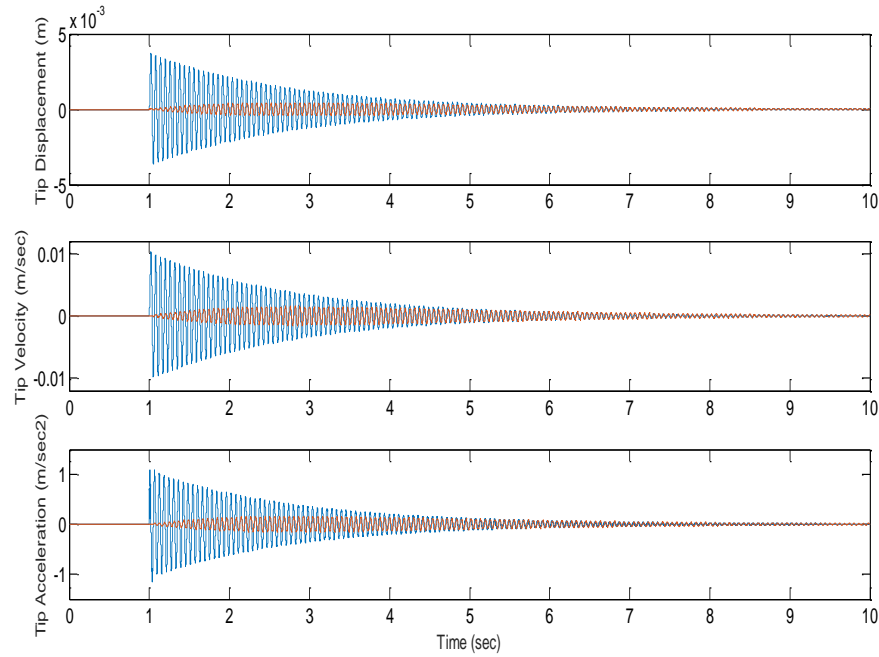


**Fig. (4) Torque of controller motor for steel cantilever beam with time.**

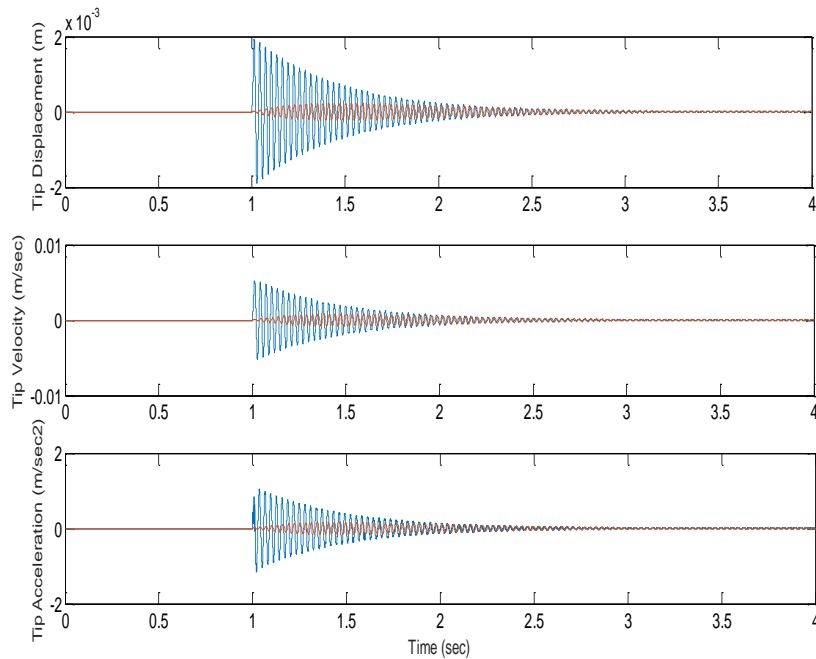


**Fig. (5) Torque of controller motor for aluminum cantilever beam with time.**





**Fig. (6) Results of steel cantilever beam where — without Controller and — with Controller.**



**Fig. (7) Result of aluminum cantilever beam where — Without Controller and — With Controller.**

## Conclusions

The present work is helpful for the vibration control for many machines, engineering structures, gadgets spacecraft's, automobiles, marine equipment's, bridges, machine tools, high rise buildings etc. The present work deals with active control of a cantilever beam vibration with a motor as controller at the fixed end. The

control is specified as displacement feedback control with the deflection for the tip of the beam providing the feedback.

It may be concluded that, the optimal controls may be applied to decrease the beam vibration within a less number of cycles. And the active control may reduce to minimum value of the vibration effect from the structure and to improve the stability. Where the overshoot reduced by 80% approximately for active case with respect of passive case. And the type of material had large effect on the vibration of the beam.

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