On ∗Soft Turing Point with Separation Axioms

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Abstract

In this paper, we use the concept of the ∗soft turing point and join it with separation axioms in soft topological space and investigate the relationship between them and study the most important properties and results of it .

Keywords: ∗Soft Turing Point, Separation Axioms.

1. Introduction and Preliminaries

The concept of soft sets was first introduced by[1] in 1999 as a general mathematical tool for dealing with uncertain objects. [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notion of soft ideal is initiated for the first time by [3]. [4] studied the soft sets theory as an analytical study and dividing the kinds to four families , to make a comparison between them and identify similarities and differences among them. [5] defined the separation axioms in soft topological space and practically in certain point of the parameters , and to study the most important properties and results of it. [6], first identified the first type of soft set . In addition, [7] and [2] defined of three types of the soft points . In this paper, we chooses one of these families to be the focus of our work is the fourth family (Simply we write the fourth family by $\boldsymbol{SS(X)}$) which is introduced by[9].We define new separation axioms in soft topological space via the concept of the ∗**Soft Turing Point** and study the most important properties and results of it.

Throughout this paper, if $(\tilde{X}_A, \tilde{\tau}, A)$ is a soft topological space and $a \in A$, $x \in X$, we say that a soft set $G_A \tilde{\in} \tilde{\tau}$ is an $a-$soft open neighborhood of x in $(\tilde{X}_A, \tilde{\tau}, A)$ if $x \in G(a)$ and we denoted by $G_{(a,x)}$ [8][D. N. Georgiou, A. C. Megaritis , 2013]. If G_A is a soft set over the universe X and $x \in X$,we say that $(a, x) \in G_A$,whenever $x \in G(a)$ for all $a \in A$ .That is for any $x \in X$, $(a, x) \notin G_A$ if $x \notin G(a)$ for some $a \in A$.

Simply we write neighborhood by (nhd).The set of all soft open nhd of a point (a, x) is denoted by $N_{\tilde{\tau}(a,x)}$

Definition 1.1 [1]
Let X be an initial universe set and A a set of parameters. A pair (F,A), where F is a map from A to P(X), is called a soft set over X.
In what follows by SS(X,A) we denote the family of all soft sets (F,A) over X.

Dentition 1.2 [10] .
We say that the soft set x_a in SS(X;A) is soft point, if for the element $a \in A$ and $x \in X$, $F(a) = \{x\}$ and $F(a') = \emptyset$ for every $a \in A$ -{a}.

Dentition 1.3 [8]
Let (F,A), (G,A) ∈ SS(X,A). We say that the pair (F,A) is a soft subset of (G,A) if F(p) ⊆ G(p), for every p ∈ A. Symbolically, we write (F,A) ⊑ (G,A). Also, we say that the pairs (F,A) and (G,A) are soft equal if (F,A) ⊑ (G,A) and (G,A) ⊑ (F,A). Symbolically, we write (F,A) = (G,A)

Definition 1.4 [11] .
A soft set F_A over χ is said to be the null soft set , denoted by $\widetilde{\Phi}_A$ if $\forall a \in A$, $F(a) = \varphi$.
A soft set F_A over χ is said to be the absolute soft set and denoted by $\tilde{\chi}_A$, if $\forall a \in A$ $F(a) = \chi$

Definition1.5 [12]
Let (F,A) ∈ SS(X,A). The soft complement of (F,A) is the soft set (H,A) ∈ SS(X,A), where the map H : A →P(X) defined as follows: H(p) = X \ F(p), for every p ∈ A. Symbolically, we write $(H,A) = (F,A)^c$

Definition 1.6 [2]
Let X be an initial universe set, A set of parameters, and $\tilde{\tau} \subseteq SS(X,A)$. We say that the family $\tilde{\tau}$ defines a soft topology on X if the following axioms are true:
(1) $\tilde{\Phi}_A$, $\tilde{X}_A$ belong to $\tilde{\tau}$
(2) If (G,A), (H,A) belong to $\tilde{\tau}$, then (G,A) ⊓ (H,A) belong to $\tilde{\tau}$
(3) If (G_i,A) belong to $\tilde{\tau}$, for every i ∈ I, then ⊔{(Gi,A) : i ∈ I} belong to $\tilde{\tau}$
The triplet (X, $\tilde{\tau}$,A) is called a soft topological space or soft space. The members of $\tilde{\tau}$ are called soft open sets in X. Also, a soft set (F,A) is called soft closed if the complement (F,A)c belongs to $\tilde{\tau}$. The family of soft closed sets is denoted by $\tilde{\tau}^c$

Definition 1.7 [13]
Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space , and let $Y \subseteq X$, the relative soft topology for $\widetilde{Y}_A$ is the collection $\tilde{\tau}_Y$ given by :
$\tilde{\tau}_Y = \{\widetilde{Y}_A \tilde{\cap} F_A , F_A \tilde{\in} \tilde{\tau}\}$. Note that $\widetilde{Y}_A$ means that $Y(a) = Y$, $\forall a \in A$.
The soft topological space $(\widetilde{Y}_A, \tilde{\tau}_Y, A)$ is called soft subspace of $(\tilde{X}_A, \tilde{\tau}, A)$.
The soft topology $\tilde{\tau}_Y$ is called induced by $\tilde{\tau}$

Definition 1.8 [13]

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , and let G_A be a soft set over the universe χ , then :

The soft closure of G_A is a soft closed set defined as :

$ClG_A = \tilde{\cap}\{S_A, S_A \text{is soft closed and } G_A \tilde{\subseteq} S_A\}$

Proposition 1.9 [7]

Let $(\tilde{\chi}_A, \tilde{\tau}, A)$ be a soft topological space , and let F_A , G_A be soft set over χ , then :

G_A is soft closed iff $Cl(G_A) = G_A$

Definition1.10 [5]

Let χ and Y be two initial universal sets and A, B be sets of parameters , $u: \chi \longrightarrow Y$ and $p: A \longrightarrow B$, then the mapping :

$f: (\chi, A) \longrightarrow (Y, A)$ (I .e $f: SS(\chi) \longrightarrow SS(Y)$) on A and B respectively is denoted by f_{pu} and can be shown as:

$$f_{pu} = \left\{\left(f_{pu}(F_A), p(A)\right), p(A) \subseteq B\right)\} \ .$$

Where : $f_{pu}(F_A)(\beta) = \begin{cases} u\left(\cup_{\alpha \in p^{-1}(\beta) \cap A \neq \varphi}\left(F(\alpha)\right)\right), & \text{if } p^{-1}(\beta) \neq \varphi \\ \varphi & \text{other wise} \end{cases}$

For $\beta \in B \ \exists \ a \in p(A)$ such that $p(a) = \beta$ **.** that is $p^{-1}(\beta) \neq \varphi$

Since $p^{-1}(\beta) \subseteq A$, hence $p^{-1}(\beta) \cap A \neq p^{-1}(\beta)$, hence we get that

$$f_{pu}(F_A)(\beta) = u\left(\bigcup_{\alpha \in p^{-1}(\beta)} F(\alpha)\right)$$

Constructing :

Since p is a mapping , so $p(A) \neq \varphi$, $\forall A \neq \varphi$, that is $\forall \beta \in p(A) \exists \ a \in A$ such that $p(a) = \beta$ and $p^{-1}(\beta) \neq \varphi$ since $a \in p^{-1}\left(p(a)\right)$ so :

$f_{pu}(F_A)(\beta) = u\{\cup_{\alpha \in p^{-1}(\beta)}\left(F(\alpha)\right)\} \ \forall \ \beta \in p(A)$.

- "if p is a one to one (1-1) , then $p^{-1}\left(p(A)\right) = A$, that is $\forall \ \beta \in p(A) \exists \ a \in A$ such that $p(a) = \beta$ and $f_{pu}(F_A)(\beta) = u\left(F(a)\right)$.
- If $G_B \in SS(Y)$ then the inverse image of G_B under f_{pu} is denoted by ${f_{pu}}^{-1}(G_B)$ is a soft set $(F_A) \in SS(\chi)$ such that

$P(a) = u^{-1}(G\left(p(\alpha)\right)$,for each $a \in A$

Remark 1.11 [5]

For each $a \in A$ and $x \in \chi$, then we can define the soft mapping f_{pu} on a $-$ soft point x_a , as follows :

1-

$$\left(f_{pu}(x_a)\right)_{p(a)} = \{(p(a), \{u(x)\})\}$$

2- Now , for $b \in B$ and $y \in Y$, ${f_{pu}}^{-1}(y_b)(a) = u^{-1}(y)$,for b=f(a)

Definition 1.12[14]

For a topological space (X, T),$x\in X$,$Y\subseteq X$, we define an ideal ${}^{Y}I_x$ respect to subspace (Y,T_Y), as follows: ${}^{Y}I_x=\{G\subseteq Y: x\in(X-G)\}$.

Definition 1.13: [3]

Let $\tilde{I}_A$ be a non-null collection of soft sets over a universe X with the same set of parameters A. Then $\tilde{I}_A\in SS(X)$ is called a soft ideal on X with the same set A if

1- $F_A\tilde{\in}\tilde{I}_A$ and $G_A\tilde{\in}\tilde{I}_A$ then $FA\tilde{\cup}G_A\tilde{\in}\tilde{I}_A$
2- $F_A\tilde{\in}\tilde{I}_A$ and $FA\tilde{\subseteq}GA$ then $G_A\tilde{\in}\tilde{I}_A$

Definition 1.14 [8] [D. N. Georgiou, A. C. Megaritis , 2013]**.**

Let (X, $\tilde{\tau}$,A) be a soft topological space, a $\in$ A, and x $\in$ X .We say that a soft set (F,A) $\in$ $\tilde{\tau}$ is an a-soft open neighborhood of x in (X, $\tilde{\tau}$,A) if x $\in$ F(a).

2- Separation Axioms using ∗Soft Turing point

Definition 2.1

A point (a,x) in A×X is called**"∗Soft turing point"** of a soft ideal **(**SI**)** in a soft topological space $(\tilde{\chi}_A,\tilde{\tau},A)$,if $(G_A)^C\in SI$, for each $G_A\tilde{\in}N_{\tilde{\tau}(a,x)}$.

Example 2.2

Let $(\tilde{\chi}_A,\tilde{\tau},A)$ be a soft topological space , $x\in X$, we define soft ideal SI (a, x), as follows: SI((a, x)) ={$G_A\tilde{\in}N_{\tilde{\tau}(a,x)}$: $(a,x)\in(G_A)^C$}. Then point (a, x) is called**"∗Soft turing point "** of SI((a, x))**.**

Remark 2.3

Let $(\tilde{X}_A,\tilde{\tau},A)$ be a soft topological space and a $\in$ A , For any pair of distinct points $x_1\neq x_2$ in X,then following properties are equivalent:

a) $(\tilde{X}_A-\{(a,x_2)\})\tilde{\in}N_{\tilde{\tau}(a,x_2)}$.
b) $(a,x_1)$is not ∗∗ soft turing point of SI (a,x_2).
c)

Proof: (a)→ (b)

Let $x_1,x_2\in X$ such that $x_1\neq x_2$. Assume that $(\tilde{X}_A-\{(a,x_2)\})\tilde{\in}N_{\tilde{\tau}(a,x_2)}$, then $\{(a,x_2)\}$ is a soft closed set in $\tilde{X}_A$,so that $\{(a,x_2)\}$=CI$\{(a,x_2)\}$.But $x_1\neq x_2$,we get that $(a,x_1)\notin$CI $\{(a,x_2)\}$ .Therefore,there exists U $\tilde{\in}N_{\tilde{\tau}(a,x_1)}$ such that, $(a,x_1)\in$ U, U $\tilde{\cap}$ $\{(a,x_2)\}=\emptyset$ .So that $(a,x_1)\in$ U, $U^c\tilde{\notin}$SI(a,x_2), because if $U^c\tilde{\in}$ SI(a,x_2),then $(a,x_2)\in$U,that means U$\tilde{\cap}$ $\{(a,x_2)\}\neq\emptyset$,this a contradiction!

Hence (a,x_1) is not ∗ soft turing point of SI(a,x_2).

(b)→ (a)

Let $x_1,x_2\in$ X such that $x_1\neq x_2$.Since $(a,x_1)$is not ∗ soft turing point of SI (a,x_2),then then there exists U $\in N_{\tilde{\tau}(a,x_1)}$ such that, $(a,x_1)\in$ U, $U^c\tilde{\notin}$SI (a,x_2), so $(a,x_2)\notin$U. Thus $(a,x_1)\in$ U, U$\tilde{\cap}$ $\{(a,x_2)\}=\emptyset$ implies $(a,x_1)\notin$CI$\{(a,x_2)\}$.Hence $\{(a,x_2)\}$= CI$\{(a,x_2)\}$ .Thus, $\{(a,x_2)\}$ is a soft closed set in $\tilde{X}_A$. Hence $(\tilde{X}_A-\{(a,x_2)\})\tilde{\in}N_{\tilde{\tau}(a,x_2)}$.

Example 2.4

Let E_X be the set of all parameters and let X be the initial universe consisting of :
$X = \{x_1, x_2\}$ and $A \tilde{\subseteq} E_X$ such that $A = \{a_1, a_2\}$.
$\tilde{\tau} = \{\tilde{\varphi}_A, \tilde{\chi}_A, G_{1_A}, G_{2_A}, G_{3_A}, G_{4_A}, G_{5_A}, G_{6_A}, G_{7_A}, G_{8_A}, G_{9_A}, G_{10_A}, G_{11_A}, G_{12_A}, G_{13_A}, G_{14_A}\}$,where

$$G_{1_A} = \{(a_1, \{x\}), (a_2, \emptyset)\}, G_{2_A} = \{(a_1, \emptyset), (a_2, \{x\})\}$$
$$G_{3_A} = \{(a_1, \emptyset), (a_2, \{y\})\}, G_{4_A} = \{(a_1, \{y\}), (a_2, \emptyset)\}$$
$$G_{5_A} = \{(a_1, \{x\}), (a_2, \{y\})\}, G_{6_A} = \{(a_1, \{y\}), (a_2, \{x\})\}$$
$$G_{7_A} = \{(a_1, \{x\}), (a_2, \{x, y\})\}, G_{8_A} = \{(a_1, \{x, y\}), (a_2, \{x\})\}$$
$$G_{9_A} = \{(a_1, \{y\}), (a_2, \{x, y\})\}, G_{10_A} = \{(a_1, \{x, y\}), (a_2, \emptyset)\}$$
$$G_{11_A} = \{(a_1, \emptyset), (a_2, \{x, y\})\}, G_{12_A} = \{(a_1, \{x, y\}), (a_2, \{y\})\}$$
$$G_{13_A} = \{(a_1, \{x\}), (a_2, \{x\})\}, G_{14_A} = \{(a_1, \{y\}), (a_2, \{y\})\}$$

Then , SI (a_1, x) $=\{\tilde{\varphi}_A, G_{2_A}, G_{3_A}, G_{4_A}, G_{6_A}, G_{9_A}, G_{11_A}, G_{14_A}\}$.
SI (a_1, y) $=\{\tilde{\varphi}_A, G_{1_A}, G_{2_A}, G_{3_A}, G_{5_A}, G_{7_A}, G_{11_A}, G_{13_A}\}$.
SI (a_2, y) $=\{\tilde{\varphi}_A, G_{1_A}, G_{2_A}, G_{4_A}, G_{6_A}, G_{8_A}, G_{10_A}, G_{13_A}\}$.
SI (a_2, x) $=\{\tilde{\varphi}_A, G_{1_A}, G_{2_A}, G_{4_A}, G_{5_A}, G_{8_A}, G_{12_A}\ G_{14_A}\}$.
(a_1, x) is $*$ soft turing point of SI$((a_1, y))$,but (a_1, y) is not $*$ soft turing point of SI (a_1, x).

Definition 2.5

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called a-SI-T_o-space if and only if, for any pair of distinct points x and y of X, (a, y) is not $*$ soft turing point of SI$((a, x))$ or (a, x) is not $*$ soft turing point of SI$((a, y))$.

Definition 2.6

The soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is called SSI-T_o-space iff $\forall\, a \in A$ the soft space $\tilde{X}_A$ is a-SI-T_o-space .

Remark 2.7

For a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$.Every SSI-$T_o$-space is a a-SI-$T_o$-space.
[Direct from definition].

Remark 2. The converse, need not be true, as seen in the following example.

Example 2. 9

Consider [Example 2.4].

Let $\tilde{\tau} = \{\tilde{\varphi}_A, \tilde{X}_A, G_{1_A}, G_{9_A}\}$ be a soft topology on $\tilde{X}_A$.Then $(\tilde{X}_A, \tilde{\tau}, A)$ is a_1-SI-T_o-space because, for any pair of distinct points x and y of X, there exist a_1- soft open set G_{9_A} contents (a_1, y) such that $(G_{9_A})^C \tilde{\notin}$ SI(a_1, x), i.e (a_1, y) is not $*$ soft turing point of SI (a_1, x) ,for some $a_1 \in A$. But it not a SSI-T_o-space , because, there exist pair of distinct points x and y of X such that (a_2, y) is $*$ soft turing point of SI (a_2, x) , (a_2, x) is $*$ soft turing point of SI (a_2, y).

Theorem 2.10

A soft subspace of a-SI-T_o– space is a-SI-T_o– space , $\forall$ a $\in$ A .

Proof : Suppose that $\tilde{Y}_A$ is a soft subspace of the of the a-SI-T_o-space $(\tilde{X}_A, \tilde{\tau}, A)$ and a $\in$ A .Let y_1 and y_2 be two distinct points of $\tilde{Y}_A$.Again, since $\tilde{X}_A$ is a-SI-T_o –space and $\tilde{Y}_A \tilde{\subseteq} \tilde{X}_A$,then (a, y_1) is not $*$ soft turing point of SI (a, y_2) or (a, y_2) is not $*$ soft turing point of SI (a, y_1).

Suppose, (a, y_1) is not $*$ soft turing point of SI (a, y_2) then there exists $U \tilde{\in} N_{\tilde{\tau}(a,y_1)}$ such that, $(a, y_1) \in U$, $U^C \tilde{\notin} SI(a, y_2)$.Then $U' = U \tilde{\cap} \tilde{Y}_A$ is $\tilde{\tau}_{Y}$- soft open contains (a, y_1) but not (a, y_2) .So that $(a, y_1) \in U'$ and $(U')^c \tilde{\notin} SI(a, y_2)$, hence $\tilde{Y}_A$ is a-SI-T_o-space.

Theorem 2.11

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_D, \tilde{\sigma}, D)$ be two soft topological spaces and let $\tilde{X}_A$ be a-SI-T_o-space ,for some a $\in$ A , if the map $f_{dv}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_D, \tilde{\sigma}, D)$ is a soft open and , d , v are onto maps , then $\tilde{Y}_D$ is d(a)-SI-T_o-space .

Proof : Let d $\in$ D and $y_1 \neq y_2$ in Y, then there exist a $\in$ A and $x_1 \neq x_2$ in X such that $v(x_1) = y_1$ and $v(x_2) = y_2$, $d(a) = b$, because d and v are onto maps .Now by assumption ,then

$(a, x_1)$is not $*$ soft turing point of $SI(a, x_2)$or $(a, x_2)$is not $*$ soft turing point of $SI(a, x_1)$, then , there exist $G_A \tilde{\in} N_{\tilde{\tau}(a,x_1)}$, $U \tilde{\in} N_{\tilde{\tau}(a,x_2)}$ such that $(a, x_1) \in G_A$,$(G_A)^C \tilde{\notin} SI(a, x_2)$ or $(a, x_2) \in U_A$,$(U_A)^C \tilde{\notin} SI(a, x_1)$.Now: $(f_{dv}(a, x_1))) \in. f_{dv}(G_A)$, $(f_{dv}(G_A))^C \tilde{\notin} SI(f_{dv}(a, x_2))$ or $(f_{dv}(a, x_2)) \in. f_{dv}(U_A)$, $(f_{dv}(U_A))^C \tilde{\notin} SI(f_{dv}(a, x_1))$, but f_{dv} is soft open ,so $f_{dv}(G_A), f_{dv}(U_A)$ are be a soft open sets in $\tilde{Y}_B$, and $(b, y_1) = (d(a), v(x_1)) = f_{dv}(a, x_1)$ and $(b, y_2) = (d(a), v(x_2)) = f_{dv}(a, x_2)$ i.e $(b, y_1)$is not $*$ soft turing point of $SI(b, y_2)$or $(b, y_2)$is not $*$ soft turing point of SI (b, y_1).Therefore , $\tilde{Y}_B$ is b-SI-T_o-space .

Theorem 2.12

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and a $\in$ A,then the following properties are equivalent:

d) $(\tilde{X}_A, \tilde{\tau}, A)$ is a-SI-T_o-space.

e) For any pair of distinct points x and y of $\tilde{X}_A$ then $Cl\{(a, x_1)\} \neq Cl\{(a, x_2)\}$.

Proof : (a)→ (**b**)

Suppose that $(\tilde{X}_A, \tilde{\tau}, A)$ is a-SI-T_o-space for some a $\in$ A and $x_1 \neq x_2$ in X , then $(a, x_1)$is not $*$ soft turing point of $SI(a, x_2)$or $(a, x_2)$is not $*$ soft turing point of $SI(a, x_1)$,so there exist $G_A \tilde{\in} N_{\tilde{\tau}(a,x_1)}$, $U \tilde{\in} N_{\tilde{\tau}(a,x_2)}$ such that $(a, x_1) \in G_A$,$(G_A)^C \tilde{\notin} SI(a, x_2)$ or $(a, x_2) \in U_A$,$(U_A)^C \tilde{\notin} SI(a, x_1)$. Then ,by Remark 2.3,then $Cl\{(a, x_1)\} = (a, x_1)$ or $Cl\{(a, x_2)\} = (a, x_2)$ That means $(a, x_1) \in Cl\{(a, x_1)\}$ and $(a, x_2) \notin Cl\{(a, x_1)\}$ or $(a, x_1) \in Cl\{(a, x_2)\}$ and $(a, x_2) \notin Cl\{(a, x_2)\}$ .Thus, $(a, x_1) \in Cl\{(a, x_1)\}$ but $(a, x_1) \notin Cl\{(a, x_2)\}$.Hence, $Cl\{(a, x_1)\} \neq Cl\{(a, x_2)\}$.

(b)→ (**a**)**:** Let a $\in$ A and $x_1 \neq x_2$ in X , with $Cl\{(a, x_1)\} \neq Cl\{(a, x_2)\}$. ,then there exist $(a, z) \in Cl\{(a, x_1)\}$, but$(a, z) \notin Cl\{(a, x_2)\}$,then $(a, x_1) \notin Cl\{(a, x_2)\}$because,if$(a, x_1) \in Cl\{(a, x_2)\}$,then $Cl\{(a, x_1)\} \tilde{\subseteq}. Cl(Cl\{(a, x_2)\}) = Cl\{(a, x_2)\}$.but $(a, z) \in Cl\{(a, x_1)\} \tilde{\subseteq}. Cl\{(a, x_2)\}$) which is a contradiction!, thus $(a, x_1) \notin Cl\{(a, x_2)\}$,which implies that $(a, x_1) \in (\tilde{X}_A - Cl\{(a, x_2)\}) \tilde{\in} N_{\tilde{\tau}(a,x_1)}$ such that $(a, x_1) \in (\tilde{X}_A - Cl\{(a, x_2)\})$, so $Cl\{(a, x_2)\} \notin SI(a, x_2)$.Hence $(\tilde{X}_A, \tilde{\tau}, A)$ is a-SI-T_o-space.

Theorem 2.13

Let $(\widetilde{Y}_B, \widetilde{\sigma}, B)$ be b-SI-T_o-spacefor $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space such that the mapping $u: X \longrightarrow Y$ be a one to one and $p: A \longrightarrow B$ be an onto map , then there exist $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is a-SI-T_o-space,if $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\widetilde{Y}_B, \widetilde{\sigma}, B)$ is a $-$soft continuous map .

Definition 2.14

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called a-SI-T_1-space if and only if, for any pair of distinct points x and y of X,(a, y) is not $*$ soft turing point of $SI((a, x))$ and (a, x) is not $*$ soft turing point of $SI((a, y))$.

Definition 2.15

The soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is called SSI-T_1-space iff $\forall\, a \in A$ the soft space $\tilde{X}_A$ is a-SI-T_1-space .

Remark 2.16

For a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$.Every SSI-$T_1$-space is a a-SI-$T_1$-space.
[Direct from definition].

Remark 2.17

The converse, need not be true, as seen in the following example.

Example 2.18

Consider [Example 2.4]
Let $\tilde{\tau} = \{\widetilde{\varphi}_A, \tilde{X}_A, G_{7_A}, G_{4_A}\}$ be a soft topology on $\tilde{X}_A$. Then $(\tilde{X}_A, \tilde{\tau}, A)$ is a_1-SI-T_1-space, but not SSI-T_1-space

Remark 2.19

Every a-SI-T_1-space is a-SI-T_o-space, but the converse is not true .

Proof Direct from [Def]. The converse, need not be true, as seen in the following example.

Example 2.20

Consider [Example 2.4]
Let $\tilde{\tau} = \{\widetilde{\varphi}_A, \tilde{X}_A, G_{13_A}\}$ be a soft topology on $\tilde{X}_A$ .Then $(\tilde{X}_A, \tilde{\tau}, A)$ is a_1-SI-T_o-space, but not a_1-SI-T_1-space

Theorem 2.21

A soft subspace of a-SI-T_1- space is a-SI-T_1- space , $\forall\, a \in A$ **.**

Proof : Similar to Theorem2.10

Theorem 2.22

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and let $\tilde{X}_A$ be a-SI-T_1-space ,for some $a \in A$, if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\widetilde{Y}_B, \widetilde{\sigma}, B)$ is a soft open and u , p are onto maps , then $\tilde{Y}_B$ is p(a)-SI-T_1-space .

Proof: Similar to Theorem2.11.

Theorem 2.23

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, $x \in X$,then the following properties are equivalent:

f) $(\tilde{X}_A, \tilde{\tau}, A)$ is a-SI-T_o-space.

g) $(\tilde{X}_A - \{(a,x)\}) \tilde{\in} N_{\tilde{\tau}(a,x)}$

Proof : Follows from Remark 2.3.

Theorem 2.24

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$,then the following properties are equivalent:

a) (X,T) is a-SI-T_1-space.

b) For any $x \neq y$ in $\tilde{X}_A$ and $a \in A$, $(\tilde{X}_A - \{(a,x)\}) \tilde{\in} N_{\tilde{\tau}(a,x)}and(\tilde{X}_A - \{(a,y)\}) \tilde{\in} N_{\tilde{\tau}(a,y)}$

Proof :[Follows from definition , Remark 2.3].

Definition2.25

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called a-SI-T_1-space if and only if, for any pair of distinct points x and y of X,(a,y) is not $*$ soft turing point of $SI((a,x))$ and (a,x) is not $*$ soft turing point of $SI((a,y))$, $SI(a,y) \cap SI(a,x) = \emptyset$.

Remark 2.27

For a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$.Every a-SI-$T_2$-space is a a-SI-$T_1$-space.But The converse, need not be true.

Example 2.29

Consider [Example 2.4]

Let $\tilde{\tau} = \{\tilde{\varphi}_A, \tilde{X}_A, G_{7_A}, G_{4_A}\}$ be a soft topology on $\tilde{X}_A$. Then $(\tilde{X}_A, \tilde{\tau}, A)$ is a_1-SI-T_1-space, but not a_1-SI-T_2-space

Theorem 2.30

Every soft subspace of a-SI-T_2-space is a-SI-T_2-space $\forall a \in A$.

Proof : Similar to Theorem2.10.

Theorem 2.31

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and let $\tilde{X}_A$ be a-SI-T_2-space ,for some $a \in A$, if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is a soft open and u , p are onto maps , then $\tilde{Y}_B$ is p(a)-SI-T_2-space .

Proof: Similar to Theorem2.11.

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نقطة التحول الطرية (*) مع بديهيات الفصل

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الخلاصة:

في هذا البحث استخدمنا مفهوم نقطة التحول الطرية (*) وربطها مع بديهيات الفصل في الفضاء التبولوجي الطري وبحث العلاقة بينهما ودراسة اهم الخصائص والنتائج لها.

الكلمات المفتاحية: بديهيات الفصل, نقطة تحول طرية (*).