

# New Type of Neutrosophic Points

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## Abstract

Through the presentation of [1] in his research about Neutrosophic set theory as well as through his definition of Neutrosophic points . We have shown that it doesn't make its sets through the union which defined by the researcher . so we need to define Neutrosophic point which achieved principle that the set represents its union points by definition an union which serve this principle as well as integration of Neutrosophic point in the Neutrosophic topological spaces and defined (interior, exterior, boundary limit and closure) points.

**Keywords:** Neutrosophic topological space

## Introduction

The notion of neutrosophic sets was by Smarandache [3].My research consists after studying the theory of Neutrosophic sets, some of the main definitions of this theory and the development of anew point that achieves the principle of union and intersection (the set represe-nts the union of its points) as well as the integration of this point in the Neutrosophic topological spaces to find the (interior , exterior and boundary) points and also to find (limited closure) points and give some examples and proposition that belong to these points .

## 1. Basic Definition and proposition in Neutrosophic set theory

### Definition 1.1[1]

Let  $X$  be a non – empty fixed sample space. A Neutrosophic crisp set denoted by (Ncs),  $w$  is an object having the form  $\dot{W} = \langle \dot{w}_1, \dot{w}_2, \dot{w}_3 \rangle$  where  $\dot{w}_1, \dot{w}_2$  and  $\dot{w}_3$  are sub set of  $X$ .

### Definition 1.2[1]

$\dot{W}$  is called a Neutrosophic crisp set if it satisfying  $\dot{w}_1 \cap \dot{w}_2 \cap \dot{w}_3 = \Phi$  and  $\dot{w}_1 \cup \dot{w}_2 \cup \dot{w}_3 = X$ .

### Example 1.3

Let  $X = \{ \hat{a}, \mathbf{b}, \mathfrak{c}, \mathfrak{d} \}$  ,  $\dot{W} = \langle \{ \hat{a}, \mathbf{b} \}, \{ \mathfrak{c} \}, \{ \mathfrak{d} \} \rangle$  where  $\dot{w}_1 = \{ \hat{a}, \mathbf{b} \}$ ,  $\dot{w}_2 = \{ \mathfrak{c} \}$  and  $\dot{w}_3 = \{ \mathfrak{d} \}$  then we have  $\dot{w}_1 \cap \dot{w}_2 \cap \dot{w}_3 = \Phi$ ,  $\{ \hat{a}, \mathbf{b} \} \cap \{ \mathfrak{c} \} \cap \{ \mathfrak{d} \} = \Phi$  and  $\dot{w}_1 \cup \dot{w}_2 \cup \dot{w}_3 = X$  ,  $\{ \hat{a}, \mathbf{b} \} \cup \{ \mathfrak{c} \} \cup \{ \mathfrak{d} \} = X$ .  $\therefore \dot{W}$  is Ncs

### Definition 1.4[1]

A Ncs of  $\Phi_N, X_N$ , in  $X$  may be define as three types :-

a-  $\Phi_N = \langle \Phi, \Phi, X \rangle$  or  $\Phi_N = \langle \emptyset, X, \Phi \rangle$  or  $\Phi_N = \langle \Phi, X, X \rangle$

b-  $X_N = \langle X, \Phi, \Phi \rangle$  or  $X_N = \langle X, X, \Phi \rangle$  or  $X_N = \langle X, \Phi, X \rangle$

**Proposition 1.5[1]**

Let  $\{\acute{W}_j, j \in \hat{J}\}$  be an arbitrary family of Neutrosophic crisp sub sets in  $X$ , then:-  
 $\acute{W}_j$  may be defined as two kinds :- 1.  $\cap$

a-  $\cap \acute{W}_j = \langle \cap \acute{w}_j_1, \cap \acute{w}_j_2, \cap \acute{w}_j_3 \rangle$  or

b-  $\cap \acute{W}_j = \langle \cap \acute{w}_j_1, \cup \acute{w}_j_2, \cup \acute{w}_j_3 \rangle$

2.  $\cup \acute{W}_j$  may be defined as two kinds :-

a-  $\cup \acute{W}_j = \langle \cup \acute{w}_j_1, \cap \acute{w}_j_2, \cap \acute{w}_j_3 \rangle$

b-  $\cup \acute{W}_j = \langle \cup \acute{w}_j_1, \cup \acute{w}_j_2, \cap \acute{w}_j_3 \rangle$

**Example 1.6**

Let  $X = \{\hat{a}, b, c\}$ ,  $\acute{w}_1 = \langle \{\hat{a}\}, \{b\}, \{c\} \rangle$

$\acute{w}_2 = \langle \{\hat{a}\}, \{c\}, \{b\} \rangle$

$\acute{w}_3 = \langle \{b\}, \{\hat{a}\}, \{c\} \rangle$

$\acute{w}_4 = \langle \{b\}, \{c\}, \{\hat{a}\} \rangle$

$\acute{w}_5 = \langle \{c\}, \{\hat{a}\}, \{b\} \rangle$

$\acute{w}_6 = \langle \{c\}, \{b\}, \{\hat{a}\} \rangle$

Then  $\cap \acute{w}_j_1 \cap \acute{w}_j_2, \cap \acute{w}_j_3 = \langle \Phi, \Phi, X \rangle = \Phi_N$  Or  $\cap \acute{w}_j_1 \cup \acute{w}_j_2, \cup \acute{w}_j_3 = \langle \Phi, X, X \rangle = \Phi_N$

**Example 1.7**

Let  $X = \{\hat{a}, b, c, \hat{d}\}$ ,  $\acute{w}_1 = \langle \{\hat{a}\}, \{b\}, \{c, \hat{d}\} \rangle$ ,  $\acute{w}_2 = \langle \{\hat{a}\}, \{c, \hat{d}\}, \{b\} \rangle$ ,  $\acute{w}_3 = \langle \{b\}, \{\hat{a}\}, \{c, \hat{d}\} \rangle$ ,  $\acute{w}_4 = \langle \{b\}, \{c, \hat{d}\}, \{\hat{a}\} \rangle$ ,  $\acute{w}_5 = \langle \{c, \hat{d}\}, \{\hat{a}\}, \{b\} \rangle$ ,  $\acute{w}_6 = \langle \{c, \hat{d}\}, \{b\}, \{\hat{a}\} \rangle$

Then we have  $\cup \acute{w}_j_1 \cap \acute{w}_j_2 \cap \acute{w}_j_3 = \langle X, \Phi, \Phi \rangle = X_N$

Or  $\cup \acute{w}_j_1 \cup \acute{w}_j_2 \cap \acute{w}_j_3 = \langle X, X, \Phi \rangle = X_N$

**Definition 1.8[1]**

Let  $X$  be anon – empty set,  $\acute{W} = \langle \acute{w}_1, \acute{w}_2, \acute{w}_3 \rangle$  if  $\acute{W}$  is Ncs, in  $X$ , then the complement of the set  $w$  is  $\acute{W}^C$  may be defined by three kinds of complement.

a)  $\acute{W}^C = \langle \acute{w}_1^c, \acute{w}_2^c, \acute{w}_3^c \rangle$  or

b)  $\acute{W}^C = \langle \acute{w}_3, \acute{w}_2, \acute{w}_1 \rangle$  or

c)  $\acute{W}^C = \langle \acute{w}_3, \acute{w}_2^c, \acute{w}_1 \rangle$

**Example 1.9**

Let  $X = \{1, 2, 3, 4, 5, 6\}$   $\acute{W} = \langle \{1, 2\}, \{3, 4\}, \{5, 6\} \rangle$  find  $\acute{W}^C$

Solution:- by definition of complement we have .

$= \langle \{3, 4, 5, 6\}, \{1, 2, 5, 6\}, \{1, 2, 3, 4\} \rangle = \acute{W}^C$

Or  $\acute{W}^C = \langle \{5, 6\}, \{3, 4\}, \{1, 2\} \rangle$

Or  $\acute{W}^C = \langle \{5, 6\}, \{1, 2, 5, 6\}, \{1, 2\} \rangle$

**Definition 1.10[1]**

Let  $X$  be anon – empty set and the Neutrosophic sets  $\acute{W}$  and  $D$  be in the form  $\acute{W} = \langle \acute{w}_1, \acute{w}_2, \acute{w}_3 \rangle$ ,  $D = \langle \hat{d}_1, \hat{d}_2, \hat{d}_3 \rangle$  we consider two possible definition for subsets ( $\acute{W} \subseteq D$ ). SO ( $\acute{W} \subseteq D$ ) may be defined as two kinds :-

1-  $\acute{W} \subseteq D \Leftrightarrow \acute{w}_1 \subseteq \hat{d}_1, \acute{w}_2 \subseteq \hat{d}_2$  and  $\acute{w}_3 \supseteq \hat{d}_3$

2-  $\acute{W} \subseteq D \Leftrightarrow \acute{w}_1 \subseteq \hat{d}_1, \acute{w}_2 \supseteq \hat{d}_2$  and  $\acute{w}_3 \supseteq \hat{d}_3$

**Definition 1.11[1]**

Let  $X$  be anon – empty set and the Ncs  $\acute{W}$  and  $D$  be of the form  $\acute{W} = \langle \acute{w}_1, \acute{w}_2, \acute{w}_3 \rangle$ ,  $D = \langle \hat{d}_1, \hat{d}_2, \hat{d}_3 \rangle$  be Ncs, then :-

1-  $\acute{W} \cap D$  may be defined as two kinds :-

a)  $\acute{W} \cap D = \langle \acute{w}_1 \cap \hat{d}_1, \acute{w}_2 \cap \hat{d}_2, \acute{w}_3 \cup \hat{d}_3 \rangle$

b)  $\acute{W} \cap D = \langle \acute{w}_1 \cap \hat{d}_1, \acute{w}_2 \cup \hat{d}_2, \acute{w}_3 \cup \hat{d}_3 \rangle$

2-  $\acute{W} \cup D$  may be defined as two kinds :-

a)  $\acute{W} \cup D = \langle \acute{w}_1 \cup \hat{d}_1, \acute{w}_2 \cap \hat{d}_2, \acute{w}_3 \cup \hat{d}_3 \rangle$

$$b) \hat{W} \cup D = \langle \hat{w}_1 \cup \hat{d}_1, \hat{w}_2 \cap \hat{d}_2, \hat{w}_3 \cap \hat{d}_3 \rangle$$

**Proposition 1.12[1,2]**

Let  $\{\hat{W}_j : j \in J\}$  be an arbitrary family of Neutrosophic crisp sub sets in  $X$ , then :-

1- $\cap \hat{W}_j$  may be defined as the following two kinds:-

$$a) \cap \hat{W}_j = \langle \cap \hat{w}_j1, \cap \hat{w}_j2, \cup \hat{w}_j3 \rangle$$

$$b) \cap \hat{W}_j = \langle \cap \hat{w}_j1, \cup \hat{w}_j2, \cup \hat{w}_j3 \rangle$$

2- $\cup \hat{W}_j$  may be defined as the following two kinds:-

$$a) \cup \hat{W}_j = \langle \cup \hat{w}_j1, \cap \hat{w}_j2, \cap \hat{w}_j3 \rangle$$

$$b) \cup \hat{W}_j = \langle \cup \hat{w}_j1, \cap \hat{w}_j2, \cup \hat{w}_j3 \rangle$$

**Example 1.13**

Let  $X = \{\hat{a}, b, c, d\}$ ,  $\hat{w}_1 = \langle \{\hat{a}, b\}, \{b\}, \{b, c\} \rangle$ , the all sub sets possible of  $\hat{W}$

$$\hat{w}_1 = \langle \{\hat{a}, b\}, \Phi, \{b, c\} \rangle$$

$$\hat{w}_2 = \langle \{\hat{a}\}, \{c\}, \{b, c\} \rangle$$

$$\hat{w}_3 = \langle \{\hat{a}\}, \{c\}, \Phi \rangle$$

$$\hat{w}_4 = \langle \{\hat{a}\}, \Phi, \{b, c\} \rangle$$

$$\hat{w}_5 = \langle \{\hat{a}, b\}, \Phi, \{c\} \rangle$$

$$\hat{w}_6 = \langle \{\hat{a}, b\}, \{c\}, \{b, c\} \rangle$$

$$\hat{w}_7 = \langle \{\hat{a}, b\}, \{c\}, \{b\} \rangle$$

$$= \langle \cup \hat{w}_j1, \cap \hat{w}_j2, \cap \hat{w}_j3 \rangle \cup \hat{W}_j$$

$$= \langle \{\hat{a}, b\}, \Phi, \Phi \rangle$$

$$\text{Or } \cup \hat{W}_j = \langle \cup \hat{w}_j1, \cap \hat{w}_j2, \cup \hat{w}_j3 \rangle$$

$$= \langle \{\hat{a}, b\}, \Phi, \{b, c\} \rangle$$

$$\text{And } \cap \hat{W}_j = \langle \cap \hat{w}_j1, \cap \hat{w}_j2, \cup \hat{w}_j3 \rangle$$

$$\hat{W}_j = \langle \Phi, \Phi, \{b, c\} \rangle \cap$$

$$= \langle \cap \hat{w}_j1, \cup \hat{w}_j2, \cup \hat{w}_j3 \rangle = \langle \Phi, \{c\}, \{b, c\} \rangle$$

From this example, we notice that the union and intersection don't achieve the concept of Neutrosophic set which acts the union of all the sub sets, then will define the union and intersection which achieve the concept of the Neutrosophic sets as follows:-

$$\hat{W}_j = \langle \cup \hat{w}_j1, \cup \hat{w}_j2, \cup \hat{w}_j3 \rangle \cup^*$$

$$\text{and } \cap_* \hat{w}_j = \langle \cap \hat{w}_j1, \cap \hat{w}_j2, \cap \hat{w}_j3 \rangle$$

$$\hat{W}_j = \langle \{\hat{a}, b\}, \{c\}, \{b, c\} \rangle \cup^*$$

$$\hat{W}_j = \langle \Phi, \Phi, \Phi \rangle \cap_*$$

**Definition 1.14[1,2]**

Let  $\hat{W} = \langle \hat{w}_1, \hat{w}_2, \hat{w}_3 \rangle$  be a Neutrosophic crisp sets  $X$ , then  $P = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$ ,  $p_1 \neq p_2 \neq p_3 \in X$  is called a neutrosophic crisp point. An Ncs,  $P = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$  belong to a Neutrosophic crisp sets  $\hat{W} = \langle \hat{w}_1, \hat{w}_2, \hat{w}_3 \rangle$  of  $X$  denoted by  $p \in \hat{W}$  may be defined by two kinds

$$a) \{p_1\} \subseteq \hat{w}_1, \{p_2\} \subseteq \hat{w}_2 \text{ and } \{p_3\} \subseteq \hat{w}_3$$

$$b) \{p_1\} \subseteq \hat{w}_1, \{p_2\} \supseteq \hat{w}_2 \text{ and } \{p_3\} \subseteq \hat{w}_3$$

**Example 1.15**

Let  $X = \{\hat{a}, b, c\}$ , let  $\hat{W} = \langle \{\hat{a}\}, \{c\}, \{b, c\} \rangle$  the all Ncp of  $\hat{W}$

$$p_1 = \langle \{\hat{a}\}, \Phi, \Phi \rangle \quad p_2 = \langle \{\hat{a}\}, \Phi, \{b\} \rangle \quad p_3 = \langle \{\hat{a}\}, \{c\}, \Phi \rangle$$

$$p_4 = \langle \{\hat{a}\}, \{c\}, \{b\} \rangle \quad p_5 = \langle \Phi, \Phi, b \rangle \quad p_6 = \langle \Phi, \{c\}, \{b\} \rangle$$

$$p_7 = \langle \Phi, \{c\}, \Phi \rangle$$

$$\{ p_1 : p_1 \in \hat{w}_1 \} \cap \{ p_2 : p_2 \in \hat{w}_2 \} \cup \{ p_3 : p_3 \in \hat{w}_3 \} \langle \cup$$

$$= \langle \{\hat{a}\}, \Phi, \{b\} \rangle$$

$$\text{Or } \langle \cup \{ p_1 : p_1 \in \hat{w}_1 \} \cap \{ p_2 : p_2 \in \hat{w}_2 \} \cup \{ p_3 : p_3 \in \hat{w}_3 \} \rangle$$

$$= \langle \{\hat{a}\}, \Phi, \Phi \rangle$$

$$\text{And } \langle \cap \{ p_1 : p_1 \in \dot{W}_1 \}, \cap \{ p_2 : p_2 \in \dot{W}_2 \}, \cup \{ p_3 : p_3 \in \dot{W}_3 \} \rangle \\ = \langle \Phi, \Phi, \{b\} \rangle$$

$$\text{Or } \langle \cap \{ p_1 : p_1 \in \dot{W}_1 \}, \cup \{ p_2 : p_2 \in \dot{W}_2 \}, \cup \{ p_3 : p_3 \in \dot{W}_3 \} \rangle = \langle \Phi, \{c\}, \{b\} \rangle$$

Throughout this example, the union and intersection also don't achieve the concept of the Neutrosophic sets which acts their union points, that's it necessary to define the union and intersection as following depending on anew point which serves his concept.

$$\cup^* P_j = \langle \cup \{ p_1 : p_1 \in \dot{W}_1 \}, \cup \{ p_2 : p_2 \in \dot{W}_2 \}, \cup \{ p_3 : p_3 \in \dot{W}_3 \} \rangle \\ \cap_* P_j = \langle \cap \{ p_1 : p_1 \in \dot{W}_1 \}, \cap \{ p_2 : p_2 \in \dot{W}_2 \}, \cap \{ p_3 : p_3 \in \dot{W}_3 \} \rangle$$

So, we can defined anew point of Neutrosophic crisp points, satisfy the concept union.

$$P_i = \Phi \begin{cases} P = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \\ \text{single point and at least one of } P_i \neq \Phi \end{cases}$$

### Example 1.16

Let  $\dot{X} = \{1, 2\}$ , let  $\dot{W} = \langle \{1\}, \{2\}, \{1,2\} \rangle$  the all points of Neutrosophic crisp  $p_1 = \langle \{1\}, \Phi, \Phi \rangle$   $p_2 = \langle \{2\}, \Phi, \Phi \rangle$   $p_3 = \langle \{1,2\}, \Phi, \Phi \rangle$   
 $p_4 = \langle \Phi, \{1\}, \Phi \rangle$   $p_5 = \langle \Phi, \{2\}, \Phi \rangle$   $p_6 = \langle \Phi, \{1,2\}, \Phi \rangle$   
 $p_7 = \langle \Phi, \Phi, \{1\} \rangle$   $p_8 = \langle \Phi, \Phi, \{2\} \rangle$   $p_9 = \langle \Phi, \Phi, \{1,2\} \rangle$   
 $P_j = \langle \cup p_{j1}, \cup p_{j2}, \cup p_{j3} \rangle \cup^*$   
 $= \langle \{1\}, \{2\}, \{1,2\} \rangle = \dot{W} = \cup \{ P : P \in \dot{W} \}$   
 $P_j = \langle \cap p_{j1}, \cap p_{j2}, \cap p_{j3} \rangle = \langle \Phi, \Phi, \Phi \rangle \cap_*$

## 2. Basic definition and proposition of neutrosophic topological space

The topology which the researcher defined it [1] is depended on the concept of the union and intersection which was defined by him, but in our definition it will be depended on the concept of the union and intersection which has been defined by us.

### Definition 2.1

A neutrosophic topology denoted by  $(N\check{T}^*)$  is a non –empty set  $\dot{X}$  and let  $\mathcal{T}^*$  is a family of neutrosophic (sub sets in  $\dot{X}$ ) satisfying the following axioms.

$$N\check{T}_1^* \quad \dot{X}_N, \Phi_N \in \mathcal{T}^* \\ N\check{T}_2^* \quad \hat{G}_1 \cap \hat{G}_2 \in \mathcal{T}^* \text{ for any } \hat{G}_1, \hat{G}_2 \in \mathcal{T}^* \\ N\check{T}_3^* \quad \cup \hat{G}_i \in \mathcal{T}^* \forall \{ \hat{G}_i, \hat{i} \in \hat{J} \} \subseteq \mathcal{T}^*$$

The pair  $(\dot{X}, \mathcal{T}^*)$  is called a Neutrosophic topological space. Denoted by  $N\check{T}^*(s)$  and any Neutrosophic set in  $\mathcal{T}^*$  is known as Neutrosophic open set denoted by (Nos) in  $\dot{X}$ . The element of  $\mathcal{T}^*$  are called open Neutrosophic sets. A Neutrosophic sets  $F$  is closed if and only it  $C(F)$  is Neutrosophic open.

### Definition 2.2

Let  $(\dot{X}, \mathcal{T}^*)$  be a Neutrosophic topological space  $\check{A} \subseteq \dot{X}$ . A point  $P \in \check{A}$  is called a Neutrosophic interior point of  $\check{A}$  if there exists a Neutrosophic open set  $\check{U} \in \mathcal{T}^*$  containing  $p_i$  such that  $p_i \in \check{U} \subseteq \check{A}$ .

The set of all Neutrosophic interior points of  $\check{A}$  is called Neutrosophic interior points of  $\check{A}$  and denoted by  $N\check{int}(\check{A})$

$$N\check{int}(\check{A}) = \{ p_i \in \check{A} : \exists \check{U} \in \mathcal{T}^* ; p_i \in \check{U} \subseteq \check{A} \}$$

**Example 2.3**

Let  $X = \{\hat{a}, b, c\}$ ,  $\mathcal{T}^* = \langle \{X_N, \Phi_N, \langle \{\hat{a}\}, \Phi, \Phi \rangle\}$   
 $\check{A} = \langle \{\hat{a}\}, \{\hat{a}\}, \{c\} \rangle$  find  $N \text{ int}(\check{A})$   
 Sol// by define of  $N \text{ int}(\check{A}) = \{ p_i \in \check{A} : \exists \check{U} \in \mathcal{T}^* ; p_i \in \check{U} \subseteq \check{A} \}$   
 $N \text{ int}(\check{A}) = \langle \{\hat{a}\}, \Phi, \Phi \rangle \therefore$

**Proposition 2.4**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space, and  $\check{A}, \check{B}$  be two Neutrosophic sets in  $X$ , then the following properties hold

- 1-  $N \text{ int}(\check{A}) \subseteq \check{A}$
- 2-  $\check{A} \subseteq \check{B} \Rightarrow N \text{ int}(\check{A}) \subseteq N \text{ int}(\check{B})$
- 3-  $\check{A} \in \mathcal{T}^*$  (i.e  $\check{A}$  is Nos)  $\Leftrightarrow N \text{ int}(\check{A}) = \check{A}$
- 4-  $N \text{ int}(\check{A}) \cap N \text{ int}(\check{B}) = N \text{ int}(\check{A} \cap \check{B})$
- 5-  $N \text{ int}(\check{A}) \cup N \text{ int}(\check{B}) \subseteq N \text{ int}(\check{A} \cup \check{B})$
- 6-  $N \text{ int}(\check{A}) = \cup \{ \check{U} \in \mathcal{T}^*, \check{U} \subseteq \check{A} \}$

Proof 1) by definition of  $N \text{ int}(\check{A})$

Proof 2) suppose that  $\check{A} \subseteq \check{B}$  to prove  $N \text{ int}(\check{A}) \subseteq N \text{ int}(\check{B})$ , let  $p_i \in N \text{ int}(\check{A}) \Rightarrow \exists \check{U} \in \mathcal{T}^*, p_i \in \check{U} \subseteq \check{A}$  (by def of  $N \text{ int}(\check{A}) \Rightarrow \exists \check{U} \in \mathcal{T}^* ; p_i \in \check{U} \subseteq \check{B}$  (since  $\check{A} \subseteq \check{B}$  )

$p_i \in N \text{ int}(\check{B})$  ( by def of  $N \text{ int}(\check{B})$  )  $\therefore N \text{ int}(\check{A}) \subseteq N \text{ int}(\check{B}) \quad \therefore$

Proof 3) & 4) & 5) and 6) obvious & clearly .

**Definition 2.5**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space and  $\check{A} \subseteq X$ . A point  $p_i \in C(\check{A})$  is called a Neutrosophic Exterior point of  $\check{A}$  if there exists a Neutrosophic open set  $\check{U} \in \mathcal{T}^*$  containing  $p_i$  such that  $p_i \in \check{U} \subseteq C(\check{A})$ . ( $C(\check{A})$  is complement of  $\check{A}$ ). The set of all Neutrosophic exterior points of  $\check{A}$  and denoted by  $N \text{ ext}(\check{A})$ ,  $N \text{ ext}(\check{A}) = \{ p_i \in C(\check{A}) ; \exists \check{U} \in \mathcal{T}^* \exists p_i \in \check{U} \subseteq C(\check{A}) \}$ .

**Example 2.6**

Let  $X = \{\hat{a}, b, c\}$ ,  $\mathcal{T}^* = \{X_N, \Phi_N, \langle \Phi, \{C\}, \Phi \rangle\}$   
 And  $\check{A} = \langle \{\hat{a}\}, \{b\}, \{b, c\} \rangle$  then  
 Sol  $C(\check{A}) = \langle \{b, c\}, \{\hat{a}, c\}, \{\hat{a}\} \rangle$  so by definition of  $N \text{ ext}(\check{A})$   
 we have  
 $N \text{ ext}(\check{A}) = \langle \Phi, \{C\}, \Phi \rangle \therefore$

**Remark 2.7**

From definition of  $N \text{ ext}(\check{A})$  we have  $N \text{ ext}(\check{A}) \subseteq C(\check{A})$  or  $N \text{ ext}(\check{A}) \cap \check{A} = \Phi$  and  $N \text{ ext}(\check{A}) = N \text{ int}(C(\check{A}))$

**Proposition 2.8**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space, and  $\check{A}, \check{B}$  be two Neutrosophic sets in  $X$ , then the following properties hold

- 1-  $N \text{ int}(\check{A}) \cap N \text{ ext}(\check{A}) = \Phi$
- 2-  $\check{A} \subseteq \check{B} \Rightarrow N \text{ ext}(\check{B}) \subseteq N \text{ ext}(\check{A})$
- 3-  $N \text{ ext}(\check{A} \cup \check{B}) = N \text{ ext}(\check{A}) \cap N \text{ ext}(\check{B})$
- 4-  $C(\check{A}) \in \mathcal{T}^*$  (i.e  $\check{A}$  closed)  $\Leftrightarrow N \text{ ext}(\check{A}) = C(\check{A})$
- 5-  $N \text{ ext}(\check{A}) \cup N \text{ ext}(\check{B}) \subseteq N \text{ ext}(\check{A} \cup \check{B})$

proof 1) by definition of  $N \text{ int}(\check{A}) \Rightarrow N \text{ int}(\check{A}) \subseteq \check{A}$  and  $N \text{ ext}(\check{A}) \subseteq C(\check{A})$

$N \text{ int}(\check{A}) \cap N \text{ ext}(\check{A}) \subseteq \Phi \Rightarrow N \text{ int}(\check{A}) \cap N \text{ ext}(\check{A}) = \Phi \Rightarrow$

proof 2 & 3 & 4 and 5 obvious

**Definition 2.9**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space  $\check{A} \subseteq X$ . A point  $p_i \in X$  is called a Neutrosophic boundary point of  $\check{A}$  if every Neutrosophic open set in  $X$  containing  $p_i$  contain at least one point of  $\check{A}$ , and least one point of  $C(\check{A})$

The set of all boundary Neutrosophic points of  $\check{A}$  is called the Neutrosophic boundary of  $\check{A}$  and denoted by  $N\check{b}d(\check{A})$

$$N\check{b}d(\check{A}) = \{ p_i \in X : \forall \check{U} \in \mathcal{T}^* \{ p_2 : p_2 \in \check{U} \}, i \in \check{U}; \check{U} \cap \check{A} \neq \Phi \wedge \check{U} \cap C(\check{A}) \neq \Phi \}$$

**Remark 2.10**

$$N\check{b}d(\check{A}) = X \setminus N\check{i}nt(\check{A}) \cap N\check{e}xt(\check{A})$$

**Proposition 2.11**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space, and  $\check{A}, \check{B}$  be two Neutrosophic sets in  $X$ , then the following properties hold

- 1-  $N\check{b}d(\check{A}) \cap N\check{i}nt(\check{A}) = \Phi$  and  $N\check{b}d(\check{A}) \cap N\check{e}xt(\check{A}) = \Phi$
- 2-  $N\check{b}d(\check{A}) = N\check{b}d(C(\check{A}))$
- 3-  $N\check{b}d(\check{A} \cup \check{B}) \subseteq N\check{b}d(\check{A}) \cup N\check{b}d(\check{B})$
- 4-  $\check{A} \in \mathcal{T}^* \Leftrightarrow N\check{b}d(\check{A}) \subseteq C(\check{A})$  or  $N\check{b}d(\check{A}) \cap \check{A} = \Phi$
- 5-  $C(\check{A}) \in \mathcal{T}^* \Leftrightarrow N\check{b}d(\check{A}) \subseteq \check{A}$  or  $N\check{b}d(\check{A}) \cap C(\check{A}) = \Phi$
- 6-  $\check{A}, C(\check{A}) \in \mathcal{T}^* \Leftrightarrow N\check{b}d(\check{A}) = \Phi$

Proof 1) Suppose that  $N\check{b}d(\check{A}) \cap N\check{i}nt(\check{A}) \neq \Phi$

$$p_i \in N\check{b}d(\check{A}) \cap N\check{i}nt(\check{A}) \Rightarrow p_i \in N\check{b}d(\check{A}) \wedge p_i \in N\check{i}nt(\check{A}) \Leftrightarrow \exists \check{U} \in \mathcal{T}^*, p_i \in \check{U} \subseteq \check{A} \text{ (def of } N\check{b}d(\check{A}))$$

$p_i \in N\check{b}d(\check{A}) \Rightarrow p_i \in C(\check{A})$  contradiction

$$N\check{b}d(\check{A}) \cap N\check{i}nt(\check{A}) = \Phi \quad \therefore$$

By similar way to prove  $N\check{b}d(\check{A}) \cap N\check{e}xt(\check{A}) = \Phi$  and proof 2 & 3 & 4 & 5 and 6 obvious & clearly

**Definition 2.12**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space  $\check{A} \subseteq X$ . A point  $p_i \in X$  is called a Neutrosophic limit point of  $\check{A}$  if every Neutrosophic open set containing  $p_i$  contains at least one point of  $\check{A}$  different from  $p_i$ .

The set of all Neutrosophic limit points of  $\check{A}$  is called the Neutrosophic derived set of  $\check{A}$  and denoted by  $N\check{l}im(\check{A}), N\check{l}im(\check{A}) = \{ p_i \in X : \forall \check{U} \in \mathcal{T}^* p_i \in \check{U} \wedge \check{U} \setminus \{p_i\} \cap \check{A} \neq \Phi \}$

**Proposition 2.13**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space, and  $\check{A}, \check{B}$  be two Neutrosophic sets in  $X$ , then the following properties hold

- 1-  $\check{A} \subseteq \check{B} \Rightarrow N\check{l}im(\check{A}) \subseteq N\check{l}im(\check{B})$
- 2-  $N\check{l}im(\check{A} \cup \check{B}) = N\check{l}im(\check{A}) \cup N\check{l}im(\check{B})$  ( The converse is not true)
- 3-  $N\check{l}im(\check{A} \cap \check{B}) \subseteq N\check{l}im(\check{A}) \cap N\check{l}im(\check{B})$  ( In general the equality isn't true )
- 4-  $C(\check{A}) \in \mathcal{T}^* \Leftrightarrow N\check{l}im(\check{A}) \subseteq \check{A}$  or  $\check{A}$  is closed  $\Leftrightarrow N\check{l}im(\check{A}) \subseteq \check{A}$

**Proof 2 :-**

$\check{A} \subseteq \check{A} \cup \check{B}$  ( by def of union )

$$N\check{l}im(\check{A}) \subseteq N\check{l}im(\check{A} \cup \check{B}) \quad \Rightarrow$$

and  $\check{B} \subseteq \check{A} \cup \check{B}$  ( by def of union )

$$N\check{l}im(\check{B}) \subseteq N\check{l}im(\check{A} \cup \check{B}) \quad \text{by part 1} \quad \therefore$$

$$N\check{l}im(\check{A}) \cup N\check{l}im(\check{B}) \subseteq N\check{l}im(\check{A} \cup \check{B}) \dots\dots\dots(1) \quad \Rightarrow$$

$$\text{Let } p_i \in N\check{l}im(\check{A}) \cup N\check{l}im(\check{B}) \Rightarrow p_i \in N\check{l}im(\check{A}) \wedge p_i \in N\check{l}im(\check{B})$$

\* ;  $p_i \in \check{U} \wedge \check{U} \setminus \{p_i\} \cap \check{A} \neq \Phi$  ( by def of Neutrosophic limit point)  $\exists \check{U} \in \mathcal{T}^*$

\* ;  $p_i \in \check{V} \wedge \check{V} \setminus \{p_i\} \cap \check{B} \neq \Phi \exists \check{V} \in \mathcal{T}^*$

\* ;  $p_i \in \check{U} \cap \check{V} \wedge (\check{U} \cap \check{V}) \setminus \{p_i\} \cap (\check{A} \cup \check{B}) \neq \Phi \therefore \check{U} \cap \check{V} \in \mathcal{T}^*$

$\bigcap_{p_i \in \mathbb{N}} \lim(\check{A} \cup B)$  ∴  
 $\mathbb{N} \lim(\check{A} \cup B) \subseteq \mathbb{N} \lim(\check{A}) \cup \mathbb{N} \lim(B) \dots \dots \dots (2)$   
 from (1) & (2) we have  $\mathbb{N} \lim(\check{A} \cup B) \subseteq \mathbb{N} \lim(\check{A}) \cup \mathbb{N} \lim(B)$   
 proof 3 & 4 are clearly

**Definition 2.15**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space  $\check{A} \subseteq X$ . the Neutrosophic closure of a set  $\check{A}$  is  $\check{A} \cup \mathbb{N} \lim(\check{A})$  and is denoted by  $\mathbb{N} cl(\check{A})$  i.e  $\mathbb{N} cl(\check{A}) = \check{A} \cup \mathbb{N} \lim(\check{A})$

**Remarket 2.16**

$$\mathbb{N} cl(\check{A}) = \bigcap \{ F \subseteq X ; C(F) \in \mathcal{T}^* \wedge \check{A} \subseteq F \}$$

**Example 2.17**

Let  $X = \{ \hat{a}, b, c \}$   $\mathcal{T}^* = \{ X_N, \Phi_N, < \Phi, \{ \hat{a}, c \}, \Phi > \}$ ,  $\check{A} = < \{ \hat{a} \}, \{ \hat{a}, b \}, \{ c \} >$   
 $F = \{ \Phi_N, X_N < X, \{ b \}, X > \}$ . So by definition of  $\mathbb{N} cl(\check{A})$  ∴  $\mathbb{N} cl(\check{A}) = \{ \cap F : F \text{ is closed, } \check{A} \subseteq F \}$  we have  
 $\mathbb{N} cl(\check{A}) = < \Phi, \Phi, \Phi >$

**Proposition 2.18**

Let  $(X, \mathcal{T}^*)$  be a Neutrosophic topological space, and  $\check{A}, B$  be two Neutrosophic sets in  $X$ , then the following properties hold

- 1-  $\check{A} \subseteq \mathbb{N} cl(\check{A})$
- 2-  $\check{A} \subseteq B \implies \mathbb{N} cl(\check{A}) \subseteq \mathbb{N} cl(B)$  ( The converse is not true )
- 3-  $\mathbb{N} cl(\check{A} \cup B) = \mathbb{N} cl(\check{A}) \cup \mathbb{N} cl(B)$
- 4-  $\mathbb{N} cl(\check{A} \cap B) \subseteq \mathbb{N} cl(\check{A}) \cap \mathbb{N} cl(B)$  ( the quality is not true true )
- 5-  $\mathbb{N} cl(\check{A}) = \bigcap \{ F \subseteq X ; C(F) \in \mathcal{T}^* \wedge \check{A} \subseteq F \}$
- 6-  $C(\check{A}) \in \mathcal{T}^*$  (i.e  $\check{A}$  is closed)  $\iff \mathbb{N} cl(\check{A}) = \check{A}$
- 7-  $\mathbb{N} cl(\mathbb{N} cl(\check{A})) = \mathbb{N} cl(\check{A})$

proof (1)  $\mathbb{N} cl(\check{A}) = \check{A} \cup \mathbb{N} \lim(\check{A})$  ( by define of  $\mathbb{N} cl(\check{A})$  )

$$\check{A} \subseteq \mathbb{N} cl(\check{A}) \implies$$

proof (2) suppose that  $\check{A} \subseteq B$  to prove  $\mathbb{N} cl(\check{A}) \subseteq \mathbb{N} cl(B)$ , since  $\check{A} \subseteq B$ ,  $\mathbb{N} \lim(\check{A}) \subseteq \mathbb{N} \lim(B)$  (Property of  $\mathbb{N} cl(\check{A})$  )

$\check{A} \subseteq B$  and  $\mathbb{N} \lim(\check{A}) \subseteq \mathbb{N} \lim(B)$

$\check{A} \cup \mathbb{N} \lim(\check{A}) \subseteq B \cup \mathbb{N} \lim(B)$

$\mathbb{N} cl(\check{A}) \subseteq \mathbb{N} cl(B)$  ( by definition of  $\mathbb{N} cl(\check{A})$  ) proof 3 & 4 & 5 & 6 and 7 is clearly.

**Conflict of Interests.**

**There are non-conflicts of interest**

**References**

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## الخلاصة

من خلال ما تم عرضه من قبل الباحث احمد سلامة لموضوعة نظرية المجموعة النتروسفكية وكذلك بتعريفه الى النقاط النتروسفكية قد بينا انها لا تكوّن مجموعتها من خلال الاتحاد الذي عرضه الباحث .  
لذا تم الحاجة الى تعريف نقطة النتروسفكية تحقق مبدأ ان المجموعة تمثل اتحاد نقاطها بتعريف اتحاد يخدم هذا المبدأ وكذلك ادماج النقطة النتروسفكية في الفضاءات التبولوجية النتروسفكية وتعريف النقاط (الداخلية , الخارجية , الحدودية) ونقاط التجمع والانغلاق .  
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