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On g-Transformation

Yusra Gatea Abbood AL-Ameri[1] , Methaq Hamza Geem[2*]

1 College of education ,Al-Qadisiyah university, Yassra-katu@gmail.com , Al-Diwaniya city, Iraq,
2 College of education ,Al-Qadisiyah university, methaq.geem@qu.edu.iq ,Al-Diwaniya city,Iraq.
*Corresponding author email: **methaq.geem@qu.edu.iq**

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Abstract:

In this paper, we introduce a new integral transformation, say g-transformation and is defined as follows:

$$G_{mn}\big(f(X)\big) = S^m \int_0^\infty e^{-s^n x} f(x)dx = F_{mn}(s)$$

Which consider generalized of most known integral transformations and also it can be find a new special integral transformations. We have studied the properties of this transformation. We have presented the general formula of g-transformation for the derivative of functions in higher orders. We also clarified the relationship between this transform and some known integral transformations such as the Laplace Transform [3], Carson transformation and Sunudu transformation [5]. We introduce new result of shifting theorem.

Key words:
Integral transformations, Differential equations, convolution theorem, Laplace transformation, g-transformations.

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حول تحويل-g

يسرى كاطع عبود[1]   ميثاق حمزة كعيم[2]

1 كلية التربية،جامعة القادسية ، قسم الرياضيات، Yassra-katu@gmail.com ، الديوانية،العراق
2 كلية التربية،جامعة القادسية ، قسم الرياضيات، Yassra-katu@gmail.com ، الديوانية،العراق

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الخلاصة:

قدمنا في هذا البحث تحويل تكاملي جديد وهو تحويل-g والمعرف كالاتي:

$$G_{mn}(f(X)) = S^m \int_0^\infty e^{-s^n x} f(x)dx = F_{mn}(s)$$

وهو تحويل معمم لكثير من التحويلات التكاملية المعروفة وكذلك يمكن إيجاد تحويلات تكاملية خاصة جديدة من الصيغة العامة لهذا التحويل.

قمنا بدراسة خواص هذا التحويل ، وقدمنا الصيغة العامة لتحويل-g لمشتقات الدوال ومن رتب عليا. كذلك وضحنا العلاقة بين هذا التحويل وبعض التحويلات التكاملية المعروفة مثل تحويل لابلاس ، تحويل كارسون وتحويل سمودو . قدمنا نتيجة جديد لنظرية الازاحة ونظرية الالتفاف وتطبيق هذا التحويل في حل المعادلات التفاضلية الاعتيادية.

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1.Introduction:

The integral transformations are one of import methods for solving many of problems. One of the first scientists who worked in integral transformation is Laplace and Fourier [7], then many integrative transformations appeared, like Al-Zughair transformation [1], Al-Zaki transformation [6], Al-Tamimi transformation [2] and Sumudu transformation [5], which had a great impact in solving differential equations with variable coefficients for certain ranks. This transformation is distinguished by the generalities of the most known integral transformations and the possibility of finding new integral transformations from it, which helps in solving differential equations with variable coefficients and in different orders.

2.1 Definition :

g-transformation $G_{mn}(f(X))$ for a function f(x) where $x \in [0,\infty[$ is defined by the following integral:

$$G_{mn}(f(X)) = S^m \int_0^\infty e^{-s^n x} f(x)dx = F_{mn}(s)$$

Such that the integral is convergent, s is positive constant.

2.2 Examples:

1- $G_{mn}(1) = S^m \int_0^\infty e^{-s^n x}\, dx$ $s^n x$

$$= \frac{s^m}{-s^n} e^{-s^n x} \Big|_0^\infty = \frac{s^m}{-s^n}(0-(1)) = \frac{s^m}{s^n} = s^{m-n}$$

2- $G_{mn}(e^{ax}) = S^m \int_0^\infty e^{-s^n x} e^{ax} dx = S^m \int_0^\infty e^{-(s^n-a)x} dx$

$$= \frac{s^m}{-(s^n-a)} e^{-(s^n-a)x} \Big|_0^\infty = \frac{s^m}{s^n-a}$$

3- $G_{mn}(x) = S^m \int_0^\infty e^{-s^n x}\, x\, dx = S^m \left[\frac{-1}{s^n} x\, e^{-s^n x} - \frac{1}{s^{2n}} e^{-s^n x}\right]_0^\infty$

$$G_{mn}(x) = S^m\left(\frac{1}{S^{2n}}\right) = S^{m-2n}$$

4- $G_{mn}(x^2) = S^m \int_0^\infty e^{-s^n x} x^2\, dx = S^m \left[\frac{-1}{s^n} x^2 e^{-s^n x} - \frac{1}{s^{2n}} x\, e^{-s^n x} - \frac{2}{s^{3n}} e^{-s^n x}\right]_0^\infty$

$$G_{mn}(x) = S^m\left(\frac{2}{S^{3n}}\right) = 2S^{m-3n}$$

2.3 Proposition:

g-transformation satisfies the linear property, that is

$$G_{mn}(af(x) \pm bg(x)) = aG_{mn}(f(x)) \pm bG_{mn}(g(x))$$

Where a,b are constants and f(x),g(x) are functions defined on $[0,\infty[$.

Proof:

$$G_{mn}(af(x) \pm bg(x)) = S^m \int_0^\infty e^{-s^n x}\left(af(x) \pm bg(x)\right)dx$$

$$= S^m \int_0^\infty e^{-s^n x}\, af(x)\, dx \pm S^m \int_0^\infty e^{-s^n x}\, bg(x)\, dx$$

$$= aS^m \int_0^\infty e^{-s^n x}\, f(x)\, dx \pm bS^m \int_0^\infty e^{-s^n x}\, g(x)\, dx$$

$$G_{mn}(af(x) \pm bg(x)) = aG_{mn}(f(x)) \pm bG_{mn}(g(x))$$

2.4 Table for g-transformation for selected functions

No.	Function, f(x)	$G_{mn}(f(X)) = S^m \int_0^\infty e^{-s^n x} f(x)dx = F_{mn}(s)$
1	K ; k constant	kS^{m-n}
2	Sin(ax)	$\frac{as^m}{s^{2n} + a^2}$
3	Cos(ax)	$\frac{s^{m+n}}{s^{2n} + a^2}$
4	Sinh(ax)	$\frac{S^m a}{(S^n - a)(S^n + a)}$
5	Cosh(ax)	$\frac{S^{m+n}}{(S^n - a)(S^n + a)}$
6	x^k	$k!\, S^{m-(k+1)n}$

2.5 Proposition:

$$G_{mn}\left(x^{-\frac{r}{k}}\right) = S^{\frac{km-n(k-r)}{k}}\, \Gamma(1 - \frac{r}{k})$$

Poof:

$$G_{mn}\left(x^{-\frac{r}{k}}\right) = S^m \int_0^\infty e^{-s^n x}\, x^{-\frac{r}{k}} dx$$

Let $y = s^n x \rightarrow x = s^{-n} y \rightarrow dx = s^{-n} dy$

$$G_{mn}\left(x^{-\frac{r}{k}}\right) = S^m \int_0^\infty e^{-y}\, (s^{-n} y)^{-\frac{r}{k}} s^{-n} dy$$

$$G_{mn}\left(x^{-\frac{r}{k}}\right) = S^{\frac{km-n(k-r)}{k}} \int_0^{\infty} y^{-\frac{r}{k}} e^{-y} dy$$

$$G_{mn}\left(x^{-\frac{r}{k}}\right) = S^{\frac{km-n(k-r)}{k}} \int_0^{\infty} y^{(1-\frac{r}{k})-1} e^{-y} dy$$

$$G_{mn}\left(x^{-\frac{r}{k}}\right) = S^{\frac{km-n(k-r)}{k}} \Gamma(1-\frac{r}{k})$$

2.6 Corollary:

$$G_{mn}\left(x^{-\frac{1}{k}}\right) = S^{\frac{km-n(k-1)}{k}} \Gamma(1-\frac{1}{k})$$

Proof:

The proof complete by using Proposition and take r=1 .

2.7 Example:

If L(f(x)) denotes to the Laplace transformation of a function f(x), then we can compute $L\left(x^{-\frac{1}{2}}\right)$ by using Corollary(2.6) with take k=2.

$$L\left(x^{-\frac{1}{2}}\right) = G_{01}\left(x^{-\frac{1}{2}}\right) = S^{\frac{0-(2-1)}{2}} \Gamma\left(1-\frac{1}{2}\right) = S^{-\frac{1}{2}} \Gamma\left(\frac{1}{2}\right) L\left(x^{-\frac{1}{2}}\right) = S^{-\frac{1}{2}}\sqrt{\pi} = \sqrt{\frac{\pi}{s}}$$

2.8 Proposition:

Given

$$G_{m1}\left(\frac{f(x)}{x+a}\right) = s^m e^{as} \int_s^{\infty} t^{-m} e^{-at} F_{m1}(t)\, dt \quad , \quad m = 0,1,2,\dots$$

Such that $F_{m1}(t) = G_{m1}(f(x))$

Proof:

Let $D = \frac{d}{ds}$

$$(D-a)G_{m1}\left(\frac{f(x)}{x+a}\right)$$
$$= ms^{m-1}\int_0^{\infty} e^{-sx}\frac{f(x)}{x+a}dx + s^m \int_0^{\infty} e^{-sx}\frac{-xf(x)}{x+a}dx - as^m \int_0^{\infty} e^{-sx}\frac{f(x)}{x+a}dx$$

$$= \frac{m}{s} G_{m1}\left(\frac{f(x)}{x+a}\right) - G_{m1}(f(x))$$

$$\left(D - (a + \frac{m}{s})\right) G_{m1}\left(\frac{f(x)}{x+a}\right) = -F_{m1}(s)$$

$$s^m e^{as} D\left(s^{-m} e^{-as} G_{m1}\left(\frac{f(x)}{x+a}\right)\right) = -F_{m1}(s)$$

$$D\left(s^{-m} e^{-as} G_{m1}\left(\frac{f(x)}{x+a}\right)\right) = -s^{-m} e^{-as} F_{m1}(s)$$

By integrating both sides we get:

$$s^{-m} e^{-as} G_{m1}\left(\frac{f(x)}{x+a}\right) = -\int_s^{\infty} t^{-m} e^{-at} F_{m1}(t)\, dt$$

$$G_{m1}\left(\frac{f(x)}{x+a}\right) = s^{-m} e^{-as} \int_s^{\infty} t^{-m} e^{-at} F_{m1}(t)\, dt$$

2.9 Proposition:

The g-transformation of the Heaviside unit step function :

$H(x-a) = \begin{cases} 0 \ ; & x \le a, \\ 1 \ ; & x > 1\,. \end{cases}$ is $G_{mn}\big(H(x-a)\big) = s^{m-n} e^{-s^n a}$

Proof:

$$G_{mn}\big(H(x-a)\big) = S^m \int_0^{\infty} e^{-s^n x}\, \mathrm{H(x-a)dx}$$

$$= S^m \int_0^a e^{-s^n x}\, 0\, \mathrm{dx} + S^m \int_a^{\infty} e^{-s^n x}\, (1)\mathrm{dx}$$

$$= \frac{s^m}{-s^n} e^{-s^n x} \Big|_a^{\infty} = \frac{s^m}{s^n} e^{-s^n a} = s^{m-n} e^{-s^n a}$$

2.10 Proposition:

1) $G_{mn}(f(x+a)) = e^{s^n a}(F_{mn}(\mathrm{s}) - S^m \int_0^a e^{-s^n t} f(t)\mathrm{dt}$

2) $G_{mn}(e^{ax} f(x)) = s^m F(s^n - a)$, F(s)=L(f(x))

Proof:

1) $G_{mn}(f(x+a)) = S^m \int_0^\infty e^{-s^n x} f(x+a)\mathrm{dx}$

Putting $\quad u = x+a \ \rightarrow x = u-a \ \ \rightarrow dx = du$

$$G_{mn}(f(x+a)) = S^m \int_a^\infty e^{-s^n(u-a)} f(u)\mathrm{du}$$

$$= S^m e^{s^n a} \int_a^\infty e^{-s^n u} f(u)\mathrm{du}$$

$$= S^m e^{s^n a}\left(\int_0^\infty e^{-s^n u} f(u)\mathrm{du} - \int_0^a e^{-s^n u} f(u)\mathrm{du}\right)$$

$$= S^m e^{s^n a} \int_0^\infty e^{-s^n u} f(u)\mathrm{du} - S^m e^{s^n a} \int_0^a e^{-s^n u} f(u)\mathrm{du}$$

$$= e^{s^n a}\left(F_{mn}(\mathrm{s}) - S^m \int_0^a e^{-s^n u} f(u)\mathrm{du}\right)$$

$$= e^{s^n a}\left(F_{mn}(\mathrm{s}) - S^m \int_0^a e^{-s^n t} f(t)\mathrm{dt}\right)$$

2-$G_{mn}(e^{ax} f(x)) = S^m \int_0^\infty e^{-s^n x} e^{ax} f(x)\mathrm{dx} = S^m \int_0^\infty e^{-x(s^n - a)} f(x)\, dx$

$$= s^m L(f(x))(s^n - a) \quad = s^m F(s^n - a)$$

2.11 Proposition:

1-$G_{mn}\left(f'(x)\right) = S^n\, G_{mn}(\mathrm{f(x)}) - S^m \mathrm{f}(0)$

2-$G_{mn}\left(f''(x)\right) = S^{2n}\, G_{mn}\left(\mathrm{f(x)}\right) - S^m[S^n \mathrm{f}(0) + \mathrm{f}'(0)]$

3-$G_{mn}\left(f^{(k)}(x)\right) = S^{kn}\, G_{mn}(\mathrm{f(x)}) - S^m \sum_0^{k-1} s^{(k-i-1)n} f^{(i)}(0)$

Proof:

1-$G_{mn}\left(f'(x)\right) = S^m \int_0^\infty e^{-s^n x} f'(x)\mathrm{dx}$

$$G_{mn}\left(f'(x)\right) = S^m\left[f(x)e^{-s^n x}\Big|_0^\infty + s^n \int_0^\infty e^{-s^n x} f(x)\mathrm{dx}\right]$$

$$G_{mn}\left(f'(x)\right) = -S^m f(0) + S^n\, G_{mn}\left(\mathrm{f(x)}\right) = S^n\, G_{mn}(\mathrm{f(x)}) - S^m \mathrm{f}(0)$$

2-By replace f' to f in (1) we get:

$$G_{mn}\left(f''(x)\right) = S^n\, G_{mn}(\mathrm{f}'(\mathrm{x})) - S^m \mathrm{f}'(0)$$

By using (1) we get

$G_{mn}(f''(x)) = S^n\,[S^n\,G_{mn}(f(x)) - S^m f(0)] - S^m f'(0)$

$G_{mn}(f''(x)) = S^{2n}\,G_{mn}(f(x)) - S^{m+n} f(0) - S^m f'(0)$

$G_{mn}(f''(x)) = S^{2n}\,G_{mn}(f(x)) - S^m[S^n f(0) + f'(0)]$

3-$G_{mn}\left(f^{(r)}(x)\right) = S^{rn}\,G_{mn}(f(x)) - S^m \sum_{i=0}^{r-1} s^{(r-i-1)n} f^{(i)}(0)$

To prove that we using mathematical induction

First:

If r=1 then by using (1) the proof completed.

Second:

We suppose that (3) is true when r=k , i.e.

$$G_{mn}\left(f^{(k)}(x)\right) = S^{kn}\,G_{mn}(f(x)) - S^m \sum_{i=0}^{k-1} s^{(k-i-1)n} f^{(i)}(0)$$

Third:

Let r=k+1

$$G_{mn}\left(f^{(k+1)}(x)\right) = S^m \int_0^\infty e^{-s^n x} f^{(k+1)} dx$$

By using partition method we get:

$$G_{mn}\left(f^{(k+1)}(x)\right) = S^m \left[e^{-s^n x} f^{(k)}(x)\Big|_0^\infty + S^n \int_0^\infty e^{-s^n x} f^{(k)} dx\right]$$

$$= -S^m f^{(k)}(0) + S^n G_{mn}\left(f^{(k)}(x)\right)$$

$$= -S^m f^{(k)}(0) + S^n\left[S^{kn}\,G_{mn}(f(x)) - S^m \sum_{i=0}^{k-1} s^{(k-i-1)n} f^{(i)}(0)\right]$$

$$= S^{(k+1)n}\,G_{mn}(f(x)) - S^m \sum_{i=0}^{k} s^{(k+1-i-1)n} f^{(i)}(0)$$

$$= S^{(k+1)n}\,G_{mn}(f(x)) - S^m \sum_{i=0}^{(k+1)-1} s^{(k+1-i-1)n} f^{(i)}(0)$$

Thus (4) is true at r=k+1

Therefore (4) is true for all positive integer number r.

2.12 Example:

Find the solution of the following equation by using g-transformation such that n=m:

$$y''(x) - 2y'(x) + 1 = 0 \ , y(0) = y'(0) = 0$$

By taking g-transformation for the equation we get:

$$s^{2n}G_{nn}(y) - 2s^n G_{nn}(y) + 1 = 0$$

$$G_{nn}(y) = \frac{-1}{s^n(s^n - 2)} = \frac{1}{2}s^{-n} + \frac{1}{4} - \frac{1}{4}\frac{s^n}{s^n - 2}$$

$$y(x) = \frac{1}{2}x + \frac{1}{4} - \frac{1}{4}e^{2x}$$

2.13 Definition:

Let $F_{mn} = G_{mn}(f(x))$ then the inverse of g-transformation denoted by:

$$G_{mn}^{-1}(F_{mn}) = f(x)$$

Where G_{mn}^{-1} returns the transform to the original function.

2.14 Remark:

If $F_{mn}^1 = G_{mn}(F_1(x)), F_{mn}^2 = G_{mn}(F_2(x)), \ldots, F_{mn}^k = G_{mn}(F_k(x))$ and $A_1, A_2, \ldots, A_k \in R$ then:

$$G_{mn}^{-1}(A_1F_{mn}^1 + A_2F_{mn}^2 + \cdots + A_kF_{mn}^k) = A_1F_1(x) + A_2F_2(x) + \cdots + A_kF_k(x)$$

2.15 Definition [3]:

Let f(x),g(x) be a functions defined on R^+ , then The convolution of the functions f(x),g(x) , is given as following:

$$(f * g)(z) = \int_0^z f(v)g(z - v)dv$$

2.16 Theorem:

Let f(x),g(x) be a functions defined on R^+ then g-transformation of the convolution (f*g) is given as the following:

$$G_{mn}\big((f * g)(x)\big) = s^{-m}G_{mn}\big(f(x)\big).G_{mn}\big(g(x)\big)$$

Proof:

$$G_{mn}\big(f(x)\big).G_{mn}\big(g(x)\big) = (S^m \int_0^\infty e^{-s^n z}\, f(z)dz)(S^m \int_0^\infty e^{-s^n y}\, g(y)dy)$$

$$= S^{2m}\int_0^{\infty}\int_0^{\infty} e^{-s^n(z+y)}\,\mathrm{f(z)g(y)dy}\,dz$$

Take x=z+y ,dy=dx

$$= S^{2m}\int_0^{\infty}\int_z^{\infty} e^{-s^n x}\,\mathrm{f(z)g(x-z)dx}\,dz$$

Since g(x-z)=0 for all x < z then:

$$= S^{2m}\int_0^{\infty}\int_0^{\infty} e^{-s^n x}\,\mathrm{f(z)g(x-z)dx}\,dz$$

By change the order of integration we get:

$$G_{mn}(f(x)).G_{mn}(g(x)) = S^{2m}\int_0^{\infty}\int_0^{\infty} e^{-s^n x}\,\mathrm{f(z)g(x-z)dz}\,dx$$

$$= S^{2m}\int_0^{\infty}\left(\left(\int_0^{x} e^{-s^n x}\,\mathrm{f(z)g(x-z)dz}\right)dx = S^{2m}\int_0^{\infty}\left(e^{-s^n x}\int_0^{x}\mathrm{f(z)g(x-z)dz}\right)dx\right.$$

$$= S^{2m}\int_0^{\infty} e^{-s^n x}(f*g)(x)\,dx = S^m G_{mn}((f*g)(x))$$

Therefore

$$G_{mn}((f*g)(x)) = S^{-m}G_{mn}(f(x)).G_{mn}(g(x))$$

2.17 Example:

$$G_{22}^{-1}\left(\frac{6s^4}{s^4+9}\right) = G_{22}^{-1}\left(s^{-2}\frac{2}{s^{-4}}\frac{3s^2}{s^4+9}\right)$$

$$= G_{22}^{-1}\left(\frac{2}{s^{-4}}\right) * G_{22}^{-1}\left(\frac{3s^2}{s^4+9}\right) = x^2 * \sin(3x)$$

2.18 Example:

$$G_{11}^{-1}(12s^{-4}) = G_{11}^{-1}(s^{-1}.6s^{-3}.2s^{1-1})$$

$$= G_{11}^{-1}(2s^{1-1}) * G_{11}^{-1}(6s^{-3}) = x^3 * 2 = \frac{x^4}{2}$$

3. The duality between g-transformation and other transformation:

3.1 Definition [3]:

The Laplace transformation of the real function f(x) , x > 0 , is defined as following:

$$L(f(x) = \int_0^\infty e^{-sx} f(x)\mathrm{dx}$$

3.2 Definition [8]:

The Laplace-Carson transformation of the real function f(x) , x > 0 , is defined as following:

$$L_C(f(x) = s\int_0^\infty e^{-sx} f(x)\mathrm{dx}$$

3.3 Definition [4]:

The Sumudu transformation of the real function f(x) , x > 0 , is defined as following:

$$S(f(x) = \frac{1}{s}\int_0^\infty e^{-\frac{1}{s}x} f(x)\mathrm{dx}$$

From the definitions of the different integral transformation , we show the dualities among the (Laplace transformation, Laplace-Carson transformation, Sumudu transformation and g- transformation, we explain it as the following:

3.4 (The duality of g- transformation and Laplace transformation):

$$L\big(f(x)\big) = G_{01}(f(x))$$

3.5 (The duality of g- transformation and Laplace-Carson transformation):

$$L_C\big(f(x)\big) = G_{11}(f(x))$$

3.6 (The duality of g- transformation and Sumudu transformation):

$$S\big(f(x)\big) = G_{-1-1}(f(x))$$

Conflict of interests.

There are non-conflicts of interest.

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