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Determine the Best and the Worst Solutions of Multi - Objective Linear Fractional Programming Problems with Interval Coefficients

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Abstract:

In this paper, the Multi-objective linear fractional programming problems with interval coefficients (MOLFPPIC) is considered. The aim of this paper is to show an iterative procedure that can be utilized to solve such problems. Questions of how to select the (best, worst) value for the objective functions, the nonlinear problem is changed into a linear programming problem (LPP), with two or more constraints and more than one varieties by two algorithms (1) subtracting the interval of numerator of the fractional from the interval of denominator and (2) the denominator to be one of the constraints. Finally, after we solve each objective function without intervals individually by modified simplex method, we use a new technique via transforming it to single-objective function with the same constraints. Numerical examples are illustrated to show the efficiency of these algorithms and new technique.

Keywords:

Linear fractional programming problems with interval coefficients, algorithms and new technique, the best and the worst value of MOLFPPIC.

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تحديد افضل الحلول واسوأها لمشكلات البرمجة الكسرية الخطية متعددة الاهداف ذو معاملات ذات فترات

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الخلاصة

في هذا البحث درسنا مشاكل البرمجة الكسرية الخطية متعددة الاهداف بمعاملات ذات فترات . الهدف من هذا البحث هو لنبين العملية التكرارية الذى يتم استخدامه لحل هذه المشاكل. استنتاجات حول كيفية تصنيف (أفضل ،أسوأ) قيمة للاهداف، يتم تحويل المشكلات غير الخطية الى مشكلة برمجة خطية، مع وجود قيدين أو أكثر من نوع واحد بواسطة خوارزميتين(١) طرح الفاصل الزمني لبسط كسري من فاصل المقام و (٢) المقام ليكون احد القيود، اخيرا بعد حل كل هدف دون فترات على حدة بطريقة السمبلكس المعدلة، نستخدم تقنية جديدة لتحويل مشاكل البرمجة الخطية متعددة الاهداف الى دالة احادية مع نفس القيود.تم توضيح بمثال عددي لإظهار كفاءة هذه الخوارزميات والتقنية الجديدة ثم مقارنة النتائج التي تم الحصول عليها للخوارزميتين بالجدول.

الكلمات الدالة

مشاكل البرمجة الكسرية الخطية متعددة الاهداف بمعاملات ذات فترات، خوازميتين وتقنية جديدة ، أفضل و أسوأ حل.

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1. INTRODUCTION

The fractional programming FP is a special case of nonlinear program, which is largely used for modeling real life problems with one or more objective (s) such as, output/employee actual cost/standard profit/cost, etc. and is applied to different disciplines such as, business, finance, engineering, economics, etc. [1]. FP is a decision problem arises to optimize the ratio subject to constraints. In real world decision situations decision maker (DM) sometimes my face to evaluate the ratio between inventory and sales, actual cost and standard cost output and employee etc., with both denomination and numerator are linear. If only one ratio is considered as an objective function under linear constraints the problem is said to be linear fractional programming (LFP) problem. Measuring relative efficiency of decision making unit in the profit sector or public. Data Envelopment Analysis (DEA)(Charnes et al.(1987); to study relative efficiency in different fields such as education ,hospital administration ,court systems , air force maintenance units , bank branches etc. are examples of application of LFP problems. Fractional programming problems have been treated in a considerable number of papers. Charnes and Cooper (1962) proved that a LFP problem may be optimized by solving two linear programming (LP) problems. Efafati and Pakaman (2012) studied an interval- value LFP problem and proved that the considered problem can be converted into an optimization problem having interval valued objective whose bounds are linear fractional functions. Hsien-Chung Wu (2008) derived the Kuhn-Tucker optimality conditions for an optimization problem with interval-valued objective function. Ammar and Kalifa (2009) dealt with LFP problem with Fuzzy parameters. Ammar and Kalifa (2004) introduced a parametric approach for solving multi- criteria linear fractional programming problem, Pandian and Jayalakshmi (2013) proposed a method for solving LFP problems, namely a denominator objective restriction method based on simplex method. Tantawy (2007,2008) brought two approaches into use to solve the LFP problem namely a feasible direction approach and duality approach. Odior (2012) brought into use an algebraic approach based on the duality concept and the partial fraction to solve the LFP problem Pandy and Punnen. (2007) introduced a procedure used an a Simplex method developed by Dantzing (1962) to solve LFP problem Mojabaet al.(2012) studied the LFP problem with interval valued in the objective function based on the Chanes and Cooper technique (1962). Dasetal. (2015) brought a note into operation for the first time on method presented by Safaei (2014)[2] . In a natural way, there is a need for generalizing the simplex technique for linear functions. All these problems are fragments of a general class of optimization problems This field of LFP was developed by Hungarian mathematician Mators [3][4][5].in 1960.Sevral method are proposed to solve this problem Charnes and cooper [6] have relied on their method depended on transforming this linear fractional is equal in value and amount to linear program [7]

In order to extend this work, we have defined MOLFPPIC and investigated several algorithms to solve LFP problems with interval coefficients. we have proposed an algorithm which depends on transforming the LFP problem to an equivalent LP problem and proposed a new approach to determine the best and the worst solution for LPIC problems finally we use a new technique to change MOLFP problems to a single objective functions.

2) Some basic definitions

a) Linear programming problem

Linear programming in math is a system process to find a maximum or minimum value of any variable in a function, it is also known by the name of optimization problem. LPP is helpful in growing complete and solving decision making problem by mathematical techniques.

The problem is widely given in a linear function which needs to be optimized subject to a set of different constraints. Majority usage of LPP is in advising the management to make the most effective and efficient use of the scares resources [10] [12]. There are many ways to solve LPP, simplex method is one of the most widely used and popular methods for linear programming. The simplex(or) modify simplex method is an iterative procedure for optaining the most feasible solution. In this method we keep transforming the value of basic variables to get maximum value for the objective function [11]

b) Linear- fractional programming

linear-fractional programming is a special case of a broader field of mathematical programming. Linear-fractional programming LFP, largely grown by Hungarian Mathematician B. Martos and his associate in the 1960's, is joined together with problems of optimization. LFP problems deal with determining the best possible allocations of a variable resources to meet certain specifications. In particular, they may deal with situations where a number of resources, such as, land, machines, materials, and people, are available and are to be combined to give way to several products. In linear- fractional programming the aim is to establish a permissible allocation of resources that will maximize or minimize some specific showing, such as profit gained per unit of cost, or cost of unit of product produced, etc. Strictly speaking, LFP study that class of mathematical programming problems in which that connection among the variables are linear, the constraint relation (the restrictions) must be in linear form and the function to be used in the best possible way (i.e. the objective function) must be a ratio of two linear functions. [8]

c) Intervals

An interval in mathematics, is as set of real numbers that contains all real numbers lying between any two numbers of the set. The basic definitions and properties, of interval numbers (or interval) and interval arithmetic.

1) A closed real interval $[x_1, x_2]$ denoted by x, is real interval number which can be defined completely by $x=[x_I, x_s] =\{ x_I \leq x \leq x_s ; x_I , x_s \in R \}$ where x_I and x_s are called infimum (or) lower bound and supremum (or) upper bound , respectively .
2) Let $x =[x_I , x_s]$ be an interval number then the midpoint is defined as $m= \frac{x_I+x_s}{2}$, satisfying the relation $x_I \leq x_m \leq x_s$ where $x_m = \frac{x_I+x_s}{2}$
3) Let $x =[x_I , x_s]$ and $y =[y_i , y_s]$ be two interval numbers then
   i) $x + y = [x_I + y_I , x_s+y_s]$
   ii) $x - y = [x_I - y_s , x_s -y_I]$. [9]

d) Interval linear fractional programming problems

The general form of LFPPIC:

Maximize (or)Minimize $Z= \frac{\sum_{j=1}^{n}[c_{jI},c_{js}]xj}{\sum_{i=1}^{m}[d_{iI},d_{is}]x_i}$……………………….(1)

Subject to: $\sum_{j=1}^{n}[a_{jI},a_{js}] \leq=\geq [g_{JI},g_{js}]$

Where i=1,….,m, j=1,………n where $x_j \in R$, c_{jI}, c_{js}, d_{iI}, d_{iS} , g_{iI}, $g_{is} \in I(R)$ is the set of all interval numbers

1) **First algorithm to find the best optimum (minimum or maximum) and the worst optimum (maximum or minimum) as follows:**

2 .1) The best minimum

Min $Z = \sum_{j=1}^{n} C_{jI}x_j$…………………………….(2)

Subject to: $\sum_{j=1}^{n} a_{j_I}x_j \geq=\leq g_{js}$

And $\sum_{i=1}^{m} d_{is}x_i \geq 1$ be one of the constraints.

2.2) The worst minimum

$$\text{Min Z} = \sum_{j=1}^{n} C_{js} x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Subject to: $\sum_{j=1}^{n} a_{js} x_j \geq = \leq g_{j_I}$

And $\sum_{i=1}^{m} d_{iI} x_i \geq 1$ be one of the constraints.

2.3) The best maximum

$$\text{Max Z} = \sum_{j=1}^{n} C_j s\, x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Subject to: $\sum_{j=1}^{n} a_{jI} x_j \geq = \leq g_{js}$

And $\sum_{i=1}^{m} d_{iI} x_i \leq 1$ be one of the constraints.

2.4) The worst maximum

$$\text{Max Z} = \sum_{j=1}^{n} C_{j_I} x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Subject to: $\sum_{j=1}^{n} a_{js} x_j \geq = \leq g_{jI}$

And $\sum_{i=1}^{m} d_{is} x_i \leq 1$ be one of the constraints.

2) Second algorithm to find the best and the worst maximum (or) minimum

Step (1) subtract the interval of numerator of the fractional into the intervals of denominator, then the linear fractional programming problems with interval coefficient transfer to linear programming problem with interval coefficients. [13]

$$\text{Maximize (or) minimize Z} = \sum_{j=1}^{n} [k_{j_I}, k_{js}] x_j \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Subject to: $\sum_{j=1}^{n}[a_{j_I}, a_{js}] \leq = \geq [g_{j_I}, g_{js}]$

Step (2) (i) The best for maximize

Maximize Z = $\sum_{j=1}^{n} k_{js}x_J$……………………….(7)

Subject to : $\sum_{j=1}^{n} a_{jI}x_j \geq = \leq g_{is}$

(ii)The worst for maximize,

Maximize Z = $\sum_{j=1}^{n} k_{jI}x_j$………………………(8)

Subject to: $\sum_{j=1}^{n} a_{js}x_j \geq = \leq g_{jI}$

Step (3) (i) The best for minimize

Minimize Z = $\sum_{j=1}^{n} k_{jI}x_j$………………………(9)

Subject to: $\sum_{j=1}^{n} a_{j_I}x_j \geq = \leq g_{js}$

(ii) The worst for minimize

Minimize Z = $\sum_{j=1}^{n} k_{js}x_J$……………………….(10)

Subject to: $\sum_{j=1}^{n} a_{js}x_j \geq = \leq g_{jI}$

Where $[k_{j_I}, k_{js}] = [c_{jI}, c_j s]$ - $[d_{iI}, d_i s]$

3. New technique to transfer multi objective to single objective functions

After solving each objective function of the MOLFPP individually by the first and second algorithms above such as:

Max. $Z_1=\mu_1$

Max. $Z_2=\mu_2$

.

.

.

Max. $Z_r=\mu_r$

Min. $Z_{r+1}=\mu_{r+1}$

.

.

.

Min. $Z_v=\mu_v$

$A_1 = \max(\mu_1, \mu_2 \dots\dots\dots, \mu_r)$, $A_2=\min(\mu_1,\dots\dots, \mu_r)$, $A_3 =\frac{A_1-A_2}{2}$

$B_1= \max(\mu_{r+1}, \dots\dots\dots\dots \mu_v)$, $B_2 =\min(\mu_{r+1},\dots\dots, \mu_v)$, $B_3 =\frac{B_1-B_2}{2}$

$$\text{Max } Z = \frac{\sum_{t=1}^{r} Maxz_t - \sum_{t=r+1}^{v} MinZ_t}{P}, \text{ where } P = \frac{A_3+B_3}{v}$$

4. Numerical Examples

The following is an example of multi objective linear fractional functions, Using the first and the second algorithm and modify simplex method to find the best and the worst solutions.

1) Max $Z_1 = \frac{[3,4]x_1+[-3,-2]x_2}{[1,2]x_1+[1,2]x_2}$

2) Max $Z_2 = \frac{[9,10]x_1+[3,4]x_2}{[1,2]x_1+[1,2]x_2}$

3) Max $Z_3 = \frac{[3,4]x_1+[-6,-5]x_2}{[0.5,1]x_1+[0.5,1]x_2}$

4) Min $Z_4 = \frac{[-6,-2]x_1+[2,11]x_2}{[0,1]x_1+[1,2]x_2}$

5) Min $Z_5 = \frac{[1,2]x_1+[0,1]x_2}{[1,2]x_1+[1,2]x_2}$

Subject to: $[1,2]x_1 + [1,2]x_2 \leq 2$

$[9,10]x_1 + [1,2]x_2 \leq 9$

$x_1, x_2 \geq 0$

5. Solution:

5.1) First algorithm

consider the following LFPPIC : 1) Max $Z_1 = \frac{[3,4]x_1+[-3,-2]x_2}{[1,2]x_1+[1,2]x_2}$

Subject to: $[1,2]x_1 + [1,2]x_2 \leq 2$

$[9,10]x_1 + [1,2]x_2 \leq 9$

$x_1, x_2 \geq 0$

- By using algorithm 3.3 .The best of objective function (1):

$$\text{Max } Z_1 = 4x_1 - 2x_2$$

Subject to:
$$x_1 + x_2 \leq 2$$
$$9x_1 + x_2 \leq 9$$
$$x_1 + x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

After solving it by modified simplex method we get the best solution = 4 at $x_1=1$, $x_2=0$

- By using algorithm 3.4.The worst of objective function (1)

$$\text{Max } Z_1 = 3x_1 - 3x_2$$

Subject to:
$$2x_1 + 2x_2 \leq 2$$
$$10x_1 + 2x_2 \leq 9$$
$$2x_1 + 2x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

After solving it by simplex method we get the worst solution =1.5 at x_1= 0.5, x_2= 0

consider the following LFPPIC : 2) Max $Z_2 = \frac{[9,10]x_1 + [3,4]x_2}{[1,2]x_1 + [1,2]x_2}$

Subject to:
$$[1,2]x_1 + [1,2]x_2 \leq 2$$
$$[9,10]x_1 + [1,2]x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

- By using algorithm 3.3.The best of objective function (2).

$$\text{Max } Z_2 = 10x_1 + 4x_2$$

Subject to:
$$x_1 + x_2 \leq 2$$
$$9x_1 + x_2 \leq 9$$
$$x_1 + x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

After solving it by simplex method we get the best solution =10 at x_1=1, x_2=0

- By using algorithm 3.4.The worst of objective function (2). Max $Z_2 = 9x_1 + 3x_2$

Subject to:
$$2x_1 + 2x_2 \leq 2$$
$$10x_1 + 2x_2 \leq 9$$
$$2x_1 + 2x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

After solving it by simplex method we get the worst solution =4.5 at x_1 ,0.5, x_2 =0

consider the following LFPPIC : 3) Max $Z_3 = \frac{[3,4]x_1 + [-6,-5]x_2}{[0{\cdot}5,1]x_1 + [0{\cdot}5,1]x_2}$

Subject to:
$$[1,2]x_1 + [1,2]x_2 \leq 2$$
$$[9,10]x_1 + [1,2]x_2 \leq 9$$
$$x_1 , x_2 \geq 0$$

- By using algorithm 3.3.The best of objective function (3).

$$\text{Max } Z_3 = 4x_1 - 5x_2$$

Subject to:
$$x_1 + x_2 \le 2$$
$$9x_1 + x_2 \le 9$$
$$0 \cdot 5x_1 + 0.5x_2 \le 1$$
$$x_1, x_2 \ge 0$$

After solving it by simplex method we get the best solution =4 at x_1 =1, x_2 =0

- By using algorithm 3.4.The worst of objective function (3).

$$\text{Max } Z_3 = 3x_1 - 6x_2$$

Subject to:
$$2x_1 + 2x_2 \le 2$$
$$10x_1 + 2x_2 \le 9$$
$$x_1 + x_2 \le 1$$
$$x_1, x_2 \ge 0$$

After solving it by simplex method we get the worst solution =2.7 at x_1 =0.9, x_2 =0

consider the following LFPPIC : 4) $\text{Min } Z_4 = \frac{[-6,-2]x_1 + [2,11]x_2}{[0,1]x_1 + [1,2]x_2}$

Subject to:
$$[1,2]x_1 + [1,2]x_2 \le 2$$
$$[9,10]x_1 + [1,2]x_2 \le 9$$
$$x_1, x_2 \ge 0$$

- By using algorithm 3.1.The best of objective function(4).

$$\text{Min } Z_4 = -6x_1 + 2x_2$$

Subject to:
$$x_1 + x_2 \le 2$$
$$9x_1 + x_2 \le 9$$
$$x_1 + 2x_2 \ge 1$$
$$x_1, x_2 \ge 0$$

After solving it by modified simplex method we get the best solution = -6 at x_1 =1, x_2 =0

- By using algorithm 3.2.The worst of objective function (4).

$$\text{Min } Z_4 = -2x_1 + 11x_2$$

$$\text{Subject to:}\quad \begin{aligned} 2x_1 + 2x_2 &\le 2 \\ 10x_1 + 2x_2 &\le 9 \\ x_2 &\ge 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

After solving it by modified simplex method we get the worst solution = 11 at $x_1 = 0$, $x_2 = 1$

consider the following LFPPIC : 5) Min $Z_5 = \frac{[1,2]x_1 + [0,1]x_2}{[1,2]x_1 + [1,2]x_2}$

$$\text{Subject to:}\quad \begin{aligned} [1,2]x_1 + [1,2]x_2 &\le 2 \\ [9,10]x_1 + [1,2]x_2 &\le 9 \\ x_1, x_2 &\ge 0 \end{aligned}$$

- By using algorithm 3.1. The best of objective function (5).

$$\text{Min } Z_5 = x_1$$

$$\text{Subject to:}\quad \begin{aligned} x_1 + x_2 &\le 2 \\ 9x_1 + x_2 &\le 9 \\ 2x_1 + 2x_2 &\ge 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

After solving it by modified simplex method we get the best solution =0 at $x_1 = 0$, $x_2 = 1$

- By using algorithm 3.2.The worst of objective function (5).

$$\text{Min } Z_5 = 2x_1 + x_2$$

$$\text{Subject to:}\quad \begin{aligned} 2x_1 + 2x_2 &\le 2 \\ 10x_1 + 2x_2 &\le 9 \\ x_1 + x_2 &\ge 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

After solving it by modified simplex method we get the worst solution = 1 at $x_1 = 0$, $x_2 = 1$

Now, in using the modified simplex method and the first algorithm, the best solutions we are obtained are given in the table (1)

Table (1)

Functions	Z_t	(x_1,x_2)	μ_t, t=1,…r,r+1…v	$\lvert\mu_t\rvert$, t=1,….r	$\lvert\mu_t\rvert$, t=r+1,….v
1	4	(1,0)	4	4	
2	10	(1,0)	10	10	
3	4	(1,0)	4	4	
4	-6	(1,0)	-6		6
5	0	(0,1)	0		0

By using new technique (4) we get:

A1=max {4,10,4} =10

A2=min {4,10,4} = 4

A3=$\frac{10-4}{2}=3$

B1=max {6,0} = 6

B2=min {6,0} = 0

B3=$\frac{6-0}{2}=3$

$$P=\frac{A_3+B_3}{v}\ P=\frac{A_3+B_3}{v}=\frac{6}{5}=1.2$$

$$\sum_{t=1}^{3}\max z_t=18x_1-3x_2\ ,\quad \sum_{t=4}^{5}Minz_i=-5x_1+2x_2$$

$$\text{Max Z}=\frac{\sum_{t=1}^{r}Maxz_t-\sum_{t=r+1}^{v}MinZ_t}{P}$$

Max Z = $\frac{23x_1-5x_2}{1.2}$ ⇔ Max Z =19.16 x_1- 4.17 x_2 we solve this objective function with constraints

Subject to:
$$x_1+x_2\le 2$$
$$9x_1+x_2\le 9$$
$$x_1,\ x_2\ge 0$$

After solving it by simplex method we get the best Solution = 19.167 at x_1 =1, x_2 =0

Now, in using the modified simplex method and the first algorithm, the worst solutions which are obtained, is given in the table (2)

Table (2)

Functions	Z_t	(x_1,x_2)	μ_t, t=1,…r,r+1,……v	$\lvert\mu_t\rvert$, t=1….r	$\lvert\mu_t\rvert$, t= r+1,…v
1	1.5	(0.5,0)	1.5	1.5	
2	4.5	(0.5,0)	4.5	4.5	
3	2.7	(0.9,0)	2.7	2.7	
4	11	(0,1)	11		11
5	1	(0,1)	1		1

By using new technique (4) we get:

A1=max {1.5,4.5,2.7} = 4.5
A2=min {1.5,4.5,2.7} = 1.5
A3=$\frac{4.5-1.5}{2} = 1.5$
B1=max {11,1} =11
B2=min {11,1} =1
B3=$\frac{11-1}{2} = 5$

$$P = \frac{A_3 + B_3}{v} = \frac{1.5+5}{5} = 1.3$$

$$\sum_{t=1}^{3} \max z_t = 15x_1 - 6x_2 \quad , \quad \sum_{t=4}^{5} Minz_t = -3x_1 + 13x_2$$

$$\text{Max Z} = \frac{\sum_{t=1}^{r} Maxz_t - \sum_{t=r+1}^{v} MinZ_t}{P}$$

$$\text{Max Z} = \frac{18x_1 - 19x_2}{1.3} \Leftrightarrow \quad \text{Max Z} = 13.85\, x_1 - 14.6\, x_2$$

We solve this objective function with constraints by simplex method:

subject to: $2x_1 + 2x_2 \le 2$

$10x_1 + 2x_2 \le 9$

$x_1, x_2 \ge 0$

We get the worst solution = 12.465 at x_1 =0.9, x_2 = 0

5.2) Second algorithm

Step (1):

1) Max $Z_1 = \frac{[3,4]x_1+[-3,-2]x_2}{[1,2]x_1+[1,2]x_2}$, Max $Z_1 = [3,4]x_1 - [1,2]x_2 + [-3,-2]x_2 - [1,2]x_2$

$= [1,3]x_1 + [-5,-3]x_2$

2) Max $Z_2 = \frac{[9,10]x_1+[3,4]x_2}{[1,2]x_1+[1,2]x_2}$, Max $Z_2 = [9,10]x_1 - [1,2]x_1 + [3,4]x_2 - [1,2]x_2$

$= [7,9]x_1 + [1,3]x_2$

3) Max $Z_3 = \frac{[3,4]x_1+[-6,-5]x_2}{[0\cdot 5,1]x_1+[0\cdot 5,1]x_2}$, Max $Z_3 = [3,4]x_1 - [0.5,1]x_1 + [-6,-5]x_2 - [0.5,1]x_2$

$= [2\cdot 5,3]x_1 + [-6\cdot 5,-6]x_2$

4) Min $Z_4 = \frac{[-6,-2]x_1+[2,11]x_2}{[0,1]x_1+[1,2]x_2}$, Min $Z_4 = [-6,-2]x_1 - [0,1]x_1 + [2,11]x_2 - [1,2]x_2$

$= [-7,-2]x_1 + [0,10]x_2$

5) Min $Z_5 = \frac{[1,2]x_1+[0,1]x_2}{[1,2]x_1+[1,2]x_2}$, Min $Z_5 = [1,2]x_1 - [1,2]x_1 + [0,1]x_2 - [1,2]x_2$

Min $Z_5 = [-1,1]x_1 + [-2,0]x_2$

Subject to: $[1,2]x_1 + [1,2]x_2 \leq 2$

$[9,10]x_1 + [1,2]x_2 \leq 9$

$x_1, x_2 \geq 0$

Step (2)

(i) The best for maximize

Maximize $Z_1 = \sum_{j=1}^{n} k_{js}x_J$,Maximize $Z_1 = 3x_1 - 3x_2$

Subject to: $\sum_{j=1}^{n} a_{jI}x_j \geq = \leq g_{is}$, $x_1 + x_2 \leq 2$, $9x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the best Solution = 3 at x_1 =1, x_2 =0

(ii) The worst for maximize

Maximize $Z=\sum_{j=1}^{n} k_{jI} x_j$, Maximize $Z_1 = x_1 - 5x_2$

Subject to: $\sum_{j=1}^{n} a_{js} x_j \geq = \leq g_{jI}$, $2x_1 + 2x_2 \leq 2$

$10x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the worst solution =0.9 at x_1 =0.9 and x_2 =0

(i) The best for maximize

Maximize $Z_2 = \sum_{j=1}^{n} k_{js} x_J$, Maximize $Z_2 = 9x_1 + 3x_2$

Subject to: $\sum_{j=1}^{n} a_{jI} x_j \geq = \leq g_{is}$, $x_1 + x_2 \leq 2$

$9x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the best Solution = 11.25 at x_1 =0.875, x_2 =1.125

(ii) The worst for maximize

Maximize$Z_2=\sum_{j=1}^{n} k_{jI} x_j$, Maximize $Z_2 = 7x_1 + 1x_2$

Subject to: $\sum_{j=1}^{n} a_{js} x_j \geq = \leq g_{jI}$, $2x_1 + 2x_2 \leq 2$

$10x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the best Solution = 6.3 at x_1 =0.9, x_2 =0

(i) The best for maximize

Maximize $Z_3 = \sum_{j=1}^{n} k_{js} x_J$, Maximize $Z_3 = 3x_1 - 6x_2$

Subject to: $\sum_{j=1}^{n} a_{jI} x_j \geq = \leq g_{is}$, $x_1 + x_2 \leq 2$, $9x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the best Solution = 3 at x_1 =1, x_2 =0

(ii)The worst for maximize

Maximize$Z_3 = \sum_{j=1}^{n} k_{jI} x_j$, Maximize $Z_3 = 2.5x_1 - 6.5x_2$

Subject to: $\sum_{j=1}^{n} a_{js} x_j \geq = \leq g_{jI}$, $2x_1 + 2x_2 \leq 2$

$10x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the worst Solution = 2.25 at x_1 =0.9 and x_2 =0

Step (3)

(i) The best for minimize

Minimize $Z_4 = \sum_{j=1}^{n} k_{jI} x_j$, Minimize $Z_4 = -7x_1$

Subject to: $\sum_{j=1}^{n} a_{j_I} x_j \geq = \leq g_{js}$, , $x_1 + x_2 \leq 2$, $9x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$

After solving it by simplex method we get the best Solution = -7 at x_1 =1 and x_2 =0

(ii) The worst for minimize

$$\text{Minimize } Z_4 = \sum_{j=1}^{n} k_{js}x_J \quad , \quad \text{Minimize } Z_4 = -2x_1+10x_2$$

$$\text{Subject to: } \sum_{j=1}^{n} a_{js}x_j \geq = \leq g_{jI} \quad , \quad 2x_1 + 2x_2 \leq 2$$

$$10x_1 + 2x_2 \leq 9$$

$$x_1 , x_2 \geq 0$$

After solving it by simplex method we get the worst solution = -1.8 at x_1 =0.9 and x_2 =0

(i) The best for minimize

$$\text{Minimize } Z_5 = \sum_{j=1}^{n} k_{jI}x_j \quad , \quad \text{Minimize } Z_5 = -x_1-2x_2$$

$$\text{Subject to: } \sum_{j=1}^{n} a_{j_I}x_j \geq = \leq g_{js}, \quad , \quad \begin{matrix} x_1 + x_2 \leq 2 \\ 9x_1 + x_2 \leq 9 \end{matrix}$$

$$x_1, x_2 \geq 0$$

After solving it by simplex method we get the best Solution = -4 at x_1 =0, x_2 =2

(ii) The worst for minimize

$$\text{Minimize } Z_5 = \sum_{j=1}^{n} k_{js}x_J \quad , \quad \text{Minimize } Z_5 = x_1$$

$$\text{Subject to: } \sum_{j=1}^{n} a_{js}x_j \geq = \leq g_{jI} \quad , \quad 2x_1 + 2x_2 \leq 2$$

$$10x_1 + 2x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

After solving it by simplex method we get the worst solution = 0 at $x_1 = 0$, $x_2 = 0$

Now, in using the modified simplex method and the second algorithm, the best solutions which are obtained, is given in the table (3):

Table (3)

functions	Z_t	(x_1,x_2)	μ_t, t=1,…r,r+1,……v	$\lvert\mu_t\rvert$, t=1….r	$\lvert\mu_t$, t=r+1,..v
1	3	(1,0)	3	3	
2	11.25	(0.875,1.125)	11.25	11.25	
3	3	(1,0)	3	3	
4	-7	(1,0)			7
5	-4	(0,2)			4

By using new technique (4) we get:

A1=max {3,11.25,3} =11.25

A2=min {3,11.25,3} = 3

$A3=\frac{11.25-3}{2} = 4.125$

B1=max {7,4} =7

B2=min {7,4} = 4, $B3=\frac{7-4}{2} = 1.5$

$P = \frac{A_3+B_3}{v} = \frac{4.125+1.5}{5} = 1.125$

$\sum_{t=1}^{3} \max z_t = 15x_1 - 6x_2$, $\sum_{t=4}^{5} Minz_t = -7.5x_1 + 1.5x_2$

$\text{Max } Z = \frac{\sum_{t=1}^{r} Maxz_t - \sum_{t=r+1}^{v} MinZ_t}{P}$, $\text{Max } Z = \frac{22.5x_1 - 7.5x_2}{1.125}$

Max Z $=20x_1 - 6.67\, x_2$

Subject to:

$x_1 + x_2 \leq 2$

$9x_1 + x_2 \leq 9$

$x_1, x_2 >= 0$

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after solving it by simplex method we get the best solution = 20 at x_1= 1 and x_2=0

Now, in using modified simplex method and the second algorithm, the worst solutions we obtained, is given in the table (4):

Table (4)

functions	Z_t	(x_1,x_2)	μ_t, t=1,…r,r+1,……v	$\lvert\mu_t\rvert$, t=1….r	$\lvert\mu_t\rvert$, t=r+1,….v
1	0.9	(0.9,0)	3	0.9	
2	6.3	(0.9,0)	6.3	6.3	
3	2.25	(0.9,0)	2.25	2.25	
4	-1.8	(0.9,0)	-1.8		1.8
5	0	(0,0)	0		0

By Using new technique (4) we get:

A1=max {0.9,6.3,2.25} =6.3

A2=min {0.9,6.3,2.25} =0.9

$A3=\frac{6.3-0.9}{2}=2.7$

B1=max {1.8,0} =1.8

B2=min {1.8,0} =0

$B3=\frac{1.8-0}{2}=0.9$

$$P=\frac{A_3+B_3}{v}=\frac{2.7+0.9}{5}=0.72$$

$$\sum_{t=1}^{3}\max z_t=10.5x_1-10.5x_2 \quad , \quad \sum_{t=4}^{5} Min\, z_t=-3x_1+10x_2$$

$$\text{Max } Z=\frac{\sum_{t=1}^{r} Maxz_t-\sum_{t=r+1}^{v} MinZ_t}{P}$$

$$\text{Max } Z=\frac{13.5x_1-20.5x_2}{0.72} \Leftrightarrow \text{Max } Z=18.75x_1-28.5x_2$$

After solving Max Z =$18.75x_1 -28.5 x_2$ with constraints

Subject to:

$$x_1+x_2 \le 2$$
$$9x_1+x_2 \le 9$$
$$x_1, x_2 \ge 0$$

by simplex method we get the worst solution = 16.875 at x_1= 0.9 and x_2=0

5.3) COMPARISION OF THE NUMERICAL RESULTS

Now, we are going to compare the numerical results which are obtained of the example as below in table (5):

Table (5): Comparison between results of the numerical example

	First algorithm	(x_1,x_2)	Second algorithm	(x_1x_2)
The best solution	19.67	(1,0)	20	(1,0)
The worst solution	12.465	(0.9,0)	16.875	(0.9,0)

6. Conclusions

In this paper, we have introduced and discussed two algorithms to get the best and the worst optimal solutions of the multi objective linear fractional programming problems with interval coefficients (MOLFPPIC), First, we change multi-objective linear fractional programming problems with interval coefficients to multi-objective linear fractional programming problems with constant coefficients. The non-linear programming problems is transformed to linear programming problem which has two or more constraints and one more varieties by two algorithms, we have used a new transformational technique for solving multi-objective linear fractional programming problems (MOLPPIC) to single objective linear programming problems with interval coefficients (SOLPPIC). Finally, after we used numerical example solved with the two different algorithms, we deduced that the value which was obtained in the both algorithms (the best and the worst) solutions are very closed.

Conflict of interests.

There are non-conflicts of interest.

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