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Free Tournaments

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Abstract

In this research, we consider finite binary relations, and study their morphology, assuming that certain isomorphism types do not occur under restriction.

Key words:

Tournament, binary relation, Bound, Morphology, Isomorphism, restriction.

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1- Introduction

A tournament Tn consists of n vertices P1, P2,..., Pn that each pair of distinct nodes Pi and Pj is Joined by one and only one of the oriented arcs

$\overrightarrow{P_i p_j}$ or $\overrightarrow{P_j p_i}$ .if the arc $P_i p_J$. is in Tn. Then we say that P_i dominates Pj .

i.e $P_i \longrightarrow P_J$

The relation of dominance thus defined is a complete, irreflexive, antisymmetric, binary relation. This method and related topics are discussed in [1]. Tournaments have also been studied in connection with sociometric relation in small groups. A survey of some of these investigations is given by [1].

2- Definitions[2]

-The relation A is embedded in the relation B iff there exists a restriction of B isomorphic to A and we note that $A \leq B$

-The finite relation A is said to be a boundary of the relation B , where A is not embedded in *B* , and all restriction of A are embedded in B.

-A binary relation T of base E is called a tournament iff it is antireflexive and for all $x,y \in E$ $T(x,y) \neq T(y,x)$ where $x \neq y$

-A Tournament T is said to be strong, if for all $x \neq y$, there exist an oriented walk from x to y. i. e $x_0 = x1$, $x2$ ….. , $xn = y$ such that $T(xi, xi+1) = +$.

-The part F of E is called an interval of R where for all $X \in E\text{-}F$ and for all $y, y^{-} \in F$: $R(y,x)$ $R = (y^{-}, x)$ and $R(x,y) = R(x, y^{-})$

-The relation R of base E is said to be decomposable if we can part E in intervals, Ei such that one at least one of them is of cardinal ≥ 2 , if this is not hold, R is said to be indecomposable.

-The relation R is expanded of the relation S and is denoted by D (s) where R is obtained from s , be replaced each point C_i of s by the set C_i is an interval of R. Note that if one of these C_i is of cardinal ≥ 2, then R is decomposable [2].

we characterize the tournaments such that A1 , A2 , A3 ,A4 and A5 are bounds of these Tournament's which defined as follows.

3- Description of Bounds Ai [3]

The Ai for the base A = (O, 1, 2, 3, 4,) is formed by expanded a 3 – cycle (cycle of 3 elements) , we obtain:

A1 : by replacing one of the point by 3 – cycle

A2 : by replacing two of the points by a chain of two elements.

A3 : by replacing two of the points by 3 – chain.

A4 : is defined by A4/ (i , i + 1 , i + 2) (i mod 5) is isomorphic to 3 – chain.

A5 : is defined by A5 / (0 , 1 ,2,3) is a positive diamond of vertex o , A5/ (0,1 ,3,4) is a negative diamond of vertex, 1 , and A5/ (2 , 4) = i.e there is an arc from 2 to 4 + The diamonds are obtained by expanded a 3-cycle one and only one of two points of the chain to two elements 1,2 (positive if the vertex is 1 and negative if the vertex is 2)

The Ai is not expanded by chain. We put B1 by the 4 – chain, B2 by the 4 – cycle B3 the positive diamond, and B4 the negative diamond.

We want to explore the tournaments which have A1, A2, A3, A4, A5, as bounds. And

A1 $\not\leq$ R , ... A5 $\not\leq$ R.

B1 $<$ R , … B4 $<$ R.

4- Description of Tournaments . M_n[4][5]

Mn is defined on the base E = (1,2…… , n) such that M (i , j) = + if and only if i < j – 1 or i = j+1 this tournaments possessed the following properties.

-Mn is isomorphic to its converse.

-Mn is indecomposable

-Mn is strong

-The number of 3-cycle which pass by the vertices 1,2,n-1 is respectively equal to 1,2,2,1 and its equal to 3 for all other vertex.

Proposition 1 B1 , B2 , B3 , and B4 embedded in M_n

Proof First Note that ∀ I , Mn/(i ,i+1 , i+2) is a 3-cycle, and these are the 3-cycles of Mn, and ∀ i , Mn/ (i, i+1 , i+2 , i+3) is a 4-cycle of Mn , then B2 ≤ Mn , and there is not any 4-cycle in Mn, then we have Mn/(1,2,3,4,5) isomorphie to the chain B1 Because this restriction does not contain any 3-cycle. Finally Mn/ (1,3,4,5) is isomorphic by B3 and Mn/ (1,2 ,3,5) is isomorphic to B4

Proposition2 A1,A2,A3,A3 , and A5 are not embedded in Mn

Proof :

We shall write the Ai by using the letters {a,b} and all the Ai are extension of 4-cycle B2

Mn/ (1,2,3,4) is a 4-cycle. To obtain A1 , one must add the element a1 to Mn/(1,2,3,4)such that

A1 (a1 , 1) = + = A1 (a1 , 3) and A1 (a1 , 2) = - = A1 (a1 , 4) we idiomatic to A1 by the word abab (each) letter of this word represents the oriented are from a1)

And for each Ai we symbolize with the following rule.

a is an arrow directed towards the vertex x , if A1 (a1 , x) = +

b if none.

The words bellow are forbidden are

abab for A1

aaba
baab
abba
babb } for A2

abaa
bbaa
bbab } for A3

aabb } for A4

aaab
baba
abbb } for A5

Now we look to the extension of 4-cycle contained in Mn.

By proposition 3, the only possible extensions are Mn (j , I , i + 1 , i + 2 , i + 3) and there are 4-types for isomorphic these extensions.

Case 1 $j \leq i-2$ which gives the word aaaa

Case 2 $j = i-1$ which gives the word baaa

Case 3 $j = i+4$ which gives the word bbba

Case 4 $j \geq i+5$ which gives the word bbbb

Each of these words are not in the previous List associated A1. Then A1 , A2 , A3 , A4 and A5 are not embedding in M_n

Theorem

Each tournament which admits A_1 , A_2 , A_3 , A_4 and A_5 as bounds is an Mn.

Proof. Let Mn of base E over 1,2,3,4…n elements, Let a $\notin E$, and Let $M_n^*$of base $E \cup \{a\}$ with the same conditions of Mn

i.e A_1,……A_5 are bounds of M_n^*

we symbolize b for the points $X \in E$ such that $M_n^*(x,c) = +$,and we symbolize that for a the points x such that $M_n^*(x,c) = +$

we know that Mn/$\{i, i+1, i+2, i+3\}$ is isomorphic to β_2 , ∀ i such that $1 \leq i \leq n-3$

1- if we take four points consequents with base of Mn and if we rely the point C to these four points , we obtain then 16 different words with the 12 words are hot allowed and four are allowed as the following:

bbba

baaa

bbbb

aaaa

The last two words belong to relation $M_n{}^*$ are not allowed and the two words bbba and baaa , by induction we shall prove that the words allowed are always of this form

For $n = 4$ evident

For $n = 5$ we get the following 8 words

bbba b

bbba a

baaa b

baaa a

bbbb a

bbbb b

aaaa b

aaaa a

The two words permint are bbbb a and baaa a for $n > 5$ the only possible for the K vertices allowed are:

bb ……… ba

ba ……… aa

bb ……… bb

aa ……… aa

If $K < n$, we extend to the right the first word by a or by b we get a not allowed word. The three other words are embedded to the right, we get for $K = 4$ the 4 words following for the word's bb ……. bb, and aa ……. aa, the only words which not associated to $M_n{}^*$ then bb ……. bba and ba ………. aaa

In the two cases $M_n{}^*$ is isomorphic to M_{n+1}

Let T be a tournament satisfy the hypotheses of a theorem

2- suppose T is of cardinal n+1 , we take M_K maximal in T , and suppose $K < n$, Card $(E - M_K) \geq 2$, or Let $r \notin M_K, r \in E$, by either b domain M_K or M_K domain r because $M_K \cup \{r\}$ is not to be M_{K+1} there for M_K is maximal

Let $D^+ = \{x: x \to M_K\}$ and $D^- = \{y: M_K \to y\}$ we obtained a 3- chain C in M_K

If $T(y,x) = +$, then $T/C \cup \{x,y\}$ isomorphic to A_3 C (contradiction) then $T(x,y) = +$ we have, for x element of D^+.

each y element of D^-, and each element z of M_K we have

$T(x,y) = + = T(x,z) = T(z,y)$, then T

Decompose in D^+ , D^- and M_K (contradiction) then $T = M_{n+1}$

And $M_n{}^* = M_n \cup \{r\} = M_{n+1}$

Conflict of interests.

There are non-conflicts of interest.

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الخلاصة

في هذا البحث قمت بدراسة العلاقات الثنائية المحدودة العناصر من حيث اشكالها بفرض استبعاد بعض العلاقات الثنائية والتي تكون اصغر منها من حيث العناصر تحت تأثير التشاكل عند تقصيرها.

الكلمات الدالة: العلاقة الدورية، العلاقة الثنائية، حدود، اشكال، تشاكل، تقصير